When do small changes matter?
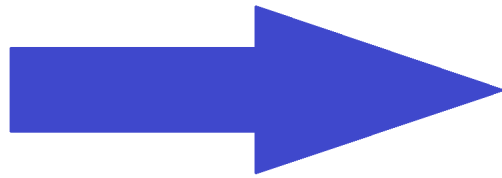
Alexandria Volkening

Math CoOp

December 9, 2014

Questions

- Will this action make a difference?
- What will the result be after a small change?
- Does it matter what kind of change it is?
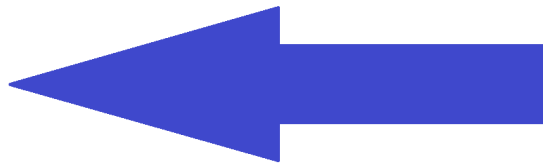- How does the answer depend on the problem?

Goal: Use mathematics to predict when small changes matter.

Three rules:

 means walk forward

 means walk backward

 means stop

Example 1

Example 1

Mathematicians call this **stable**.
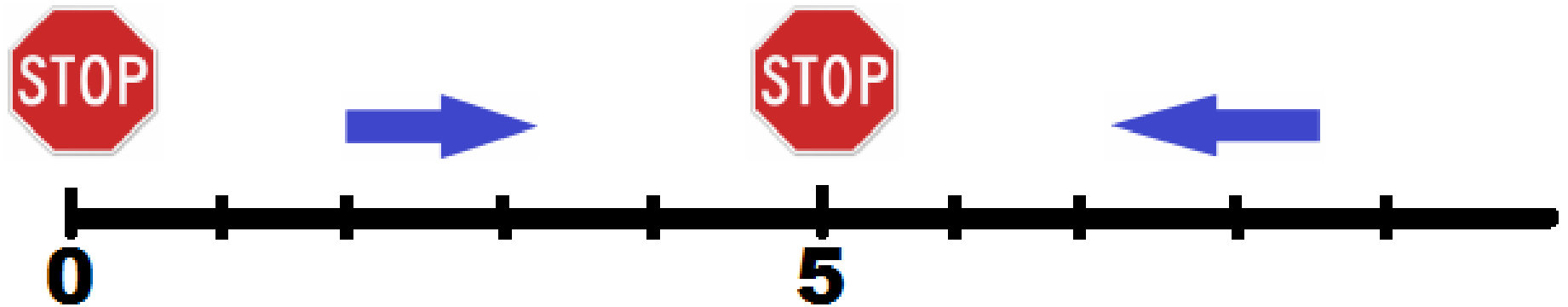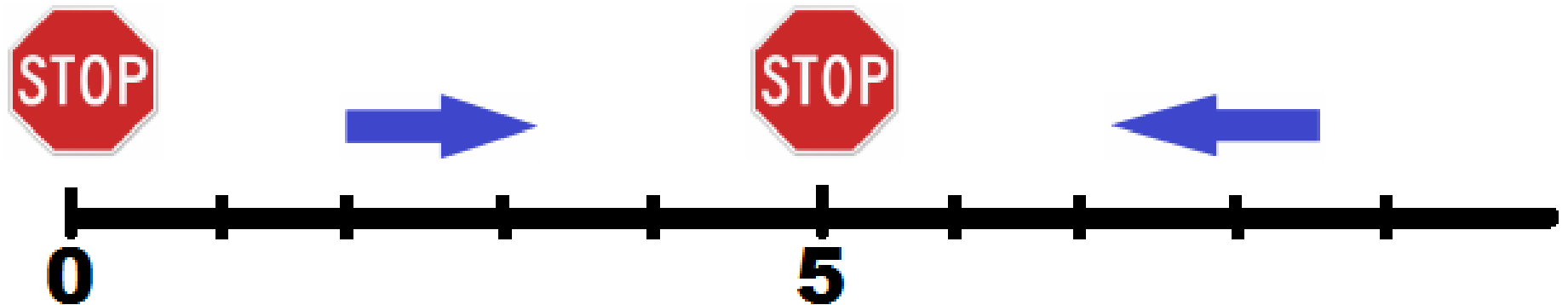
Example 2

Example 2

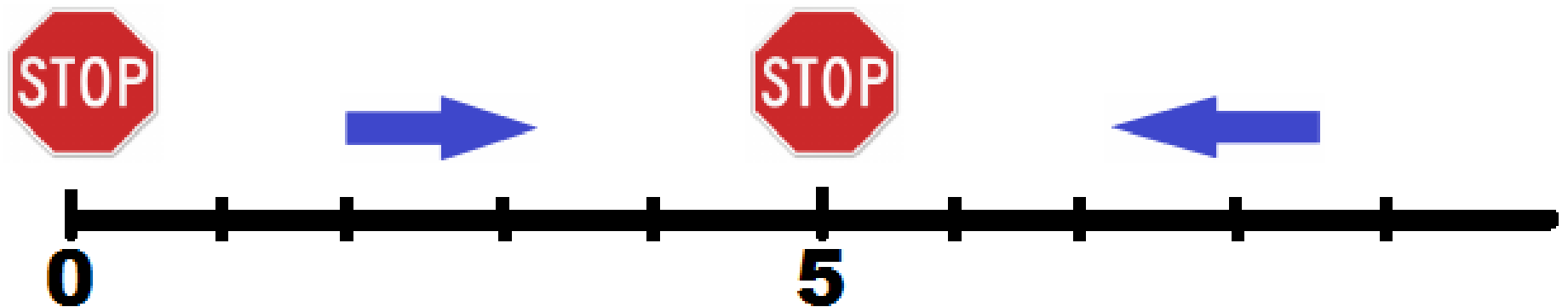Mathematicians call this **unstable**.

Example 3

Example 3

This set of rules is related to **population growth**.
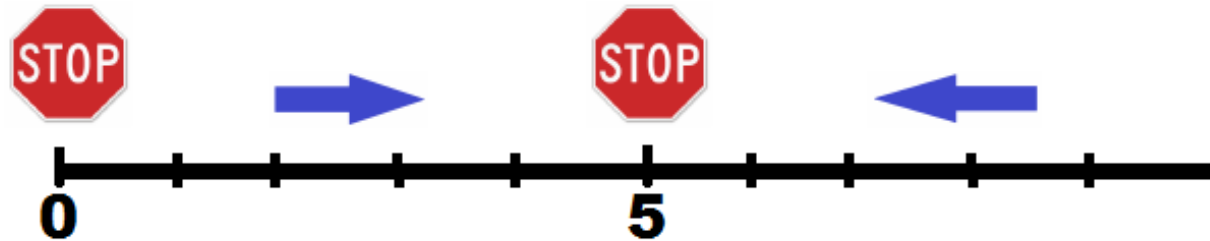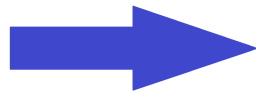It describes the **logistic equation**.

Example 3

This set of rules is related to **population growth**.
It describes the **logistic equation**.

New problem: how can we use this to describe population growth?
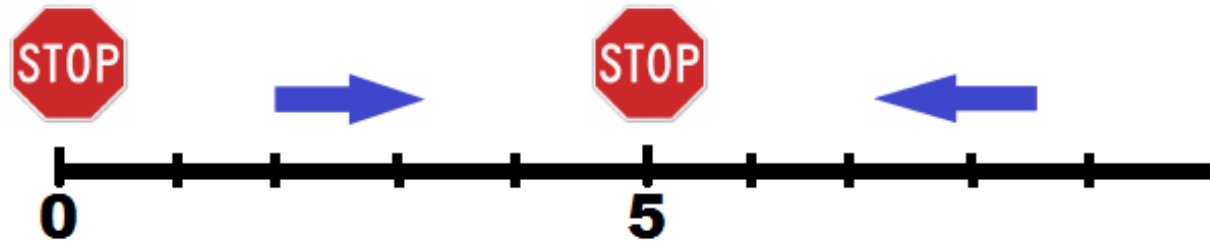
Population growth

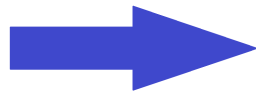New way of thinking about our old directions:

 means population increases
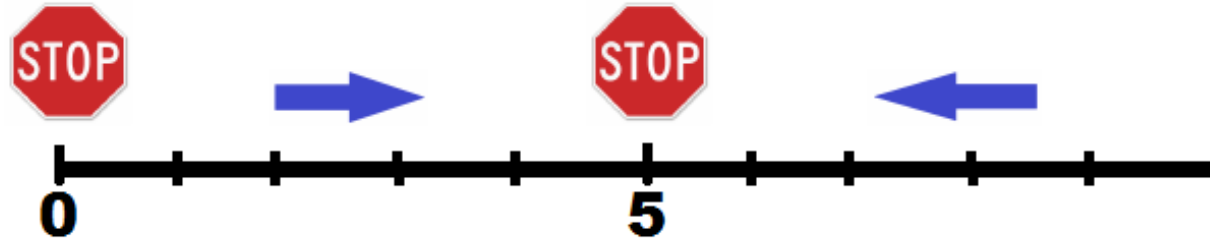
Population growth

New way of thinking about our old directions:

 means population increases (positive change)

Population growth

New way of thinking about our old directions:

        means population increases (positive change)

        means population decreases

Population growth

New way of thinking about our old directions:

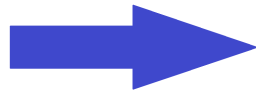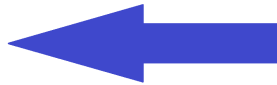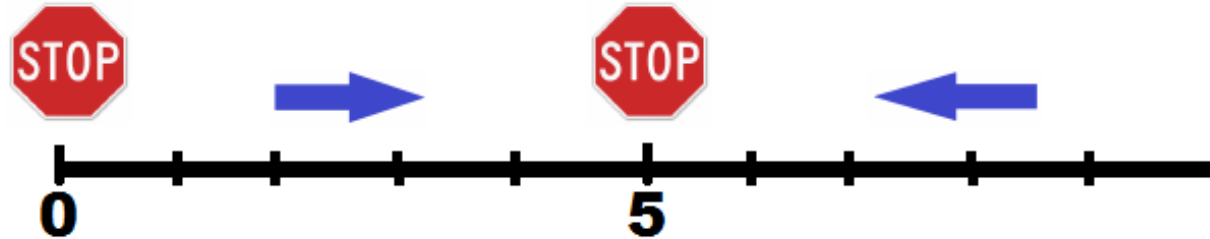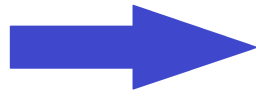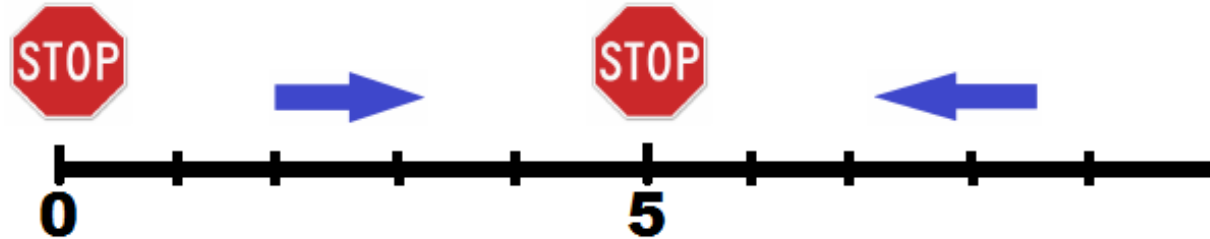 means population increases (positive change)

 means population decreases (negative change)

Population growth

New way of thinking about our old directions:

 means population increases (positive change)

 means population decreases (negative change)

 means population stays the same

Population growth

New way of thinking about our old directions:
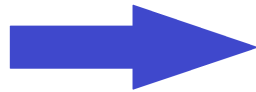
      means population increases (positive change)
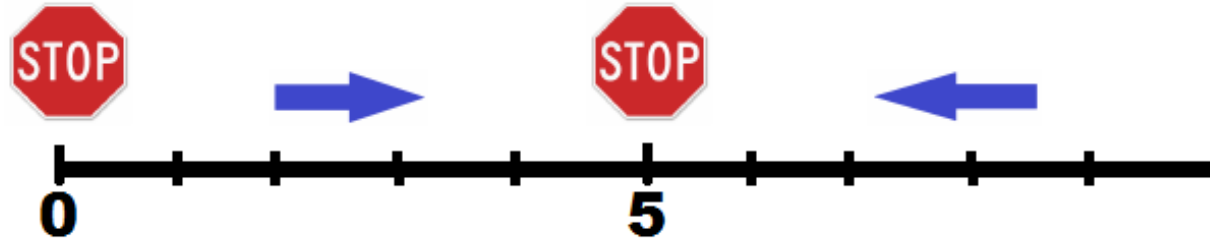
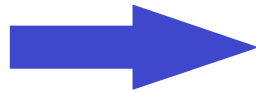   means population decreases (negative change)
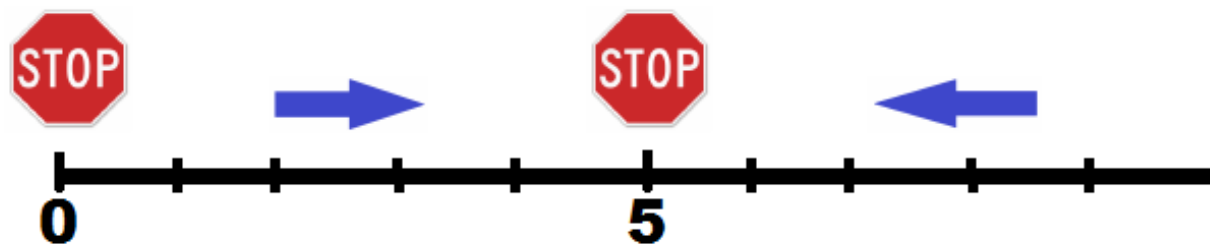
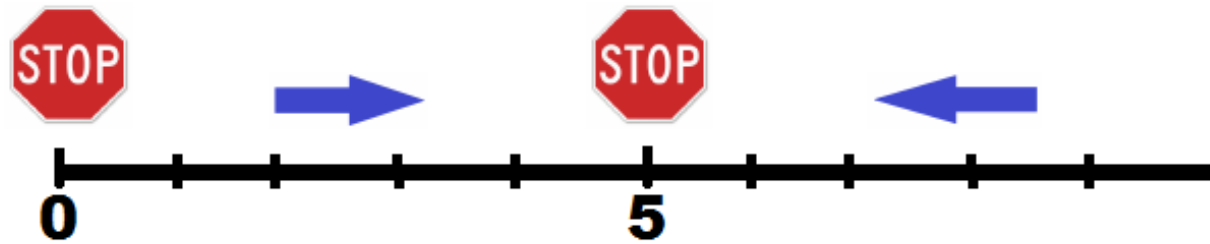   means population stays the same (zero change)

Finding a mathematical rule

We can describe this picture using rules:

1. Put the population size in the box $\Box$

Finding a mathematical rule

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1. Put the population size in the box

2. Calculate

population change $= \boxed{} \times (5 - \boxed{}) \div 5$
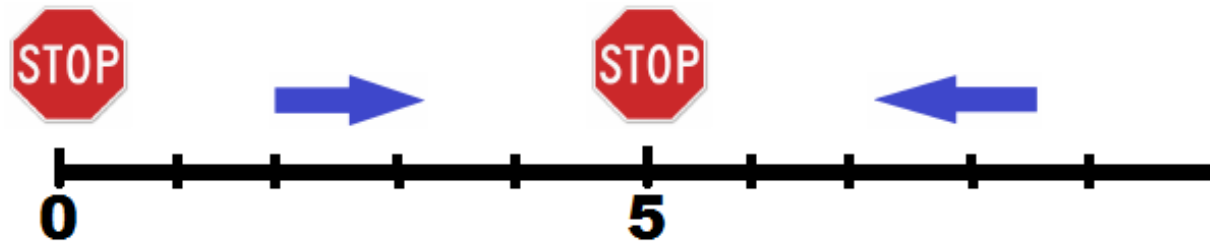
Finding a mathematical rule
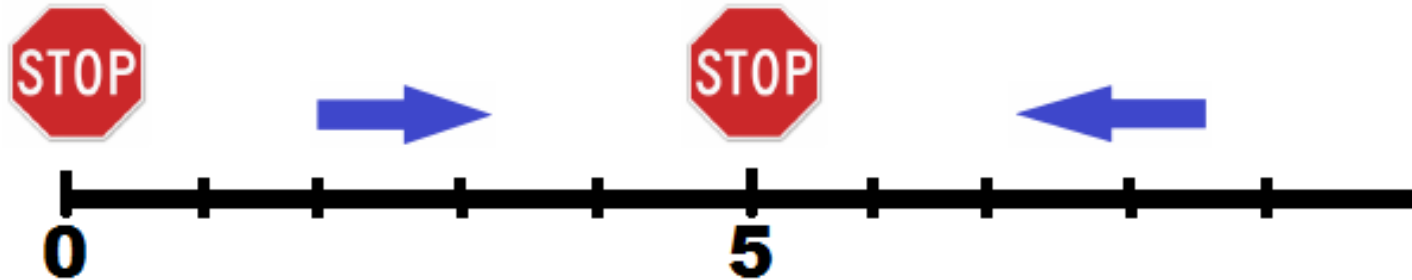

We can describe this picture using rules:

1. Put the population size in the box $\Box$

2. Calculate

   population change $= \Box \times (5 - \Box) \div 5$

3. Positive means →, negative means ←, and zero means STOP

Mathematical rule for population growth

is the same thing as the logistic equation
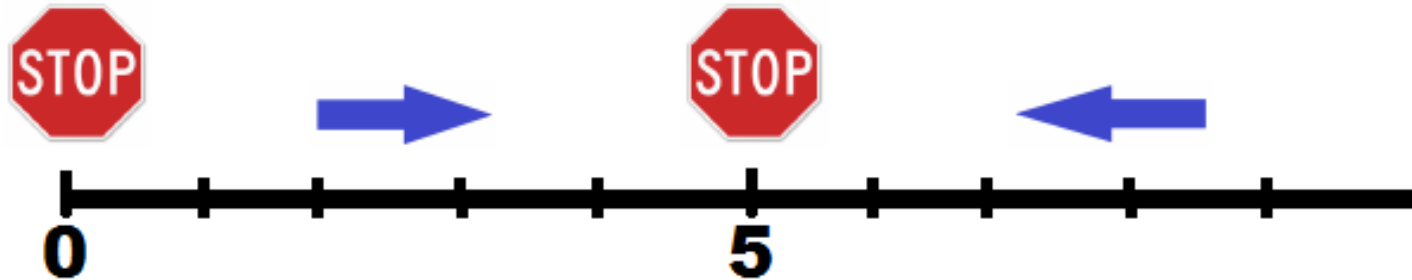
population change = population $\times$ (5 - population) $\div$ 5

Mathematical rule for population growth

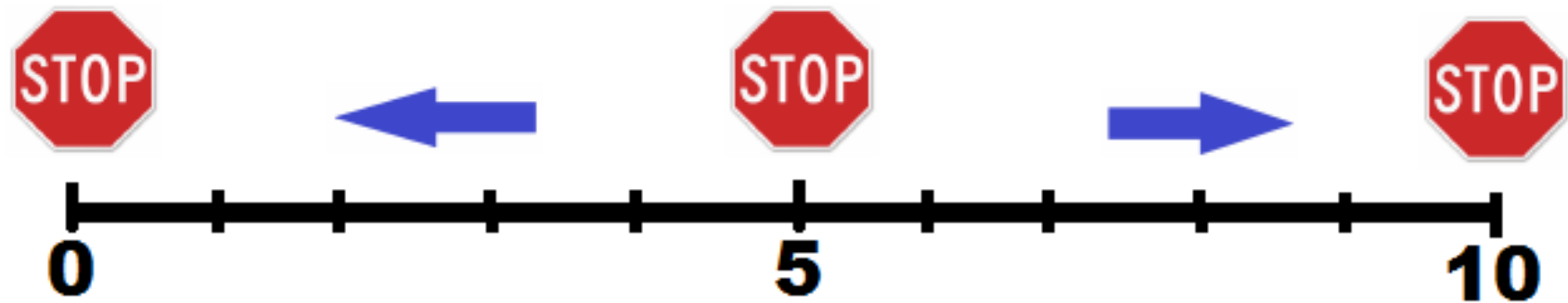
is the same thing as the logistic equation

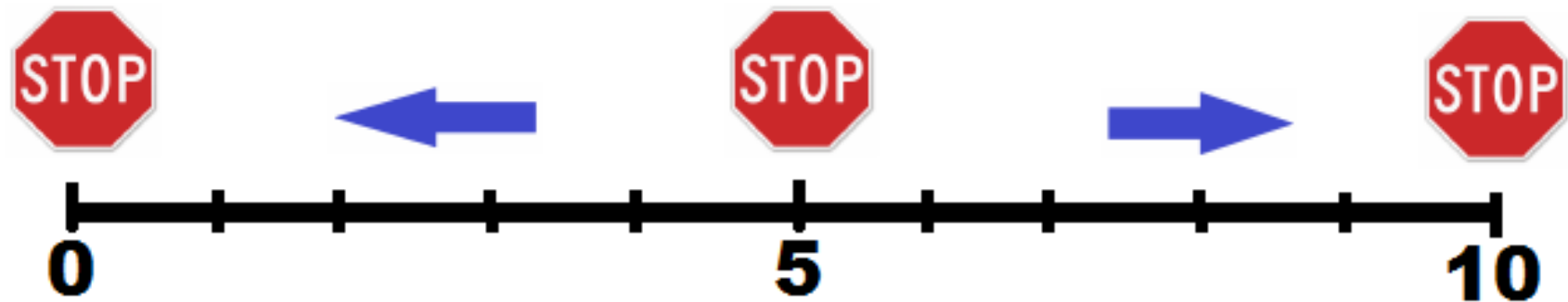population change = population × (5 - population) ÷ 5

This describes the growth of stable populations. Small changes don't make a big difference.

What about extinction?
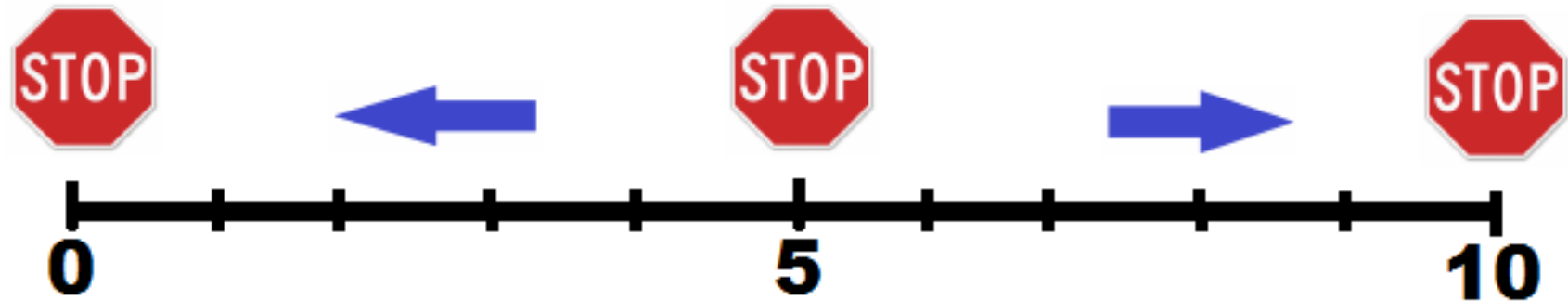
What about extinction?

Here a small change can make a huge difference - extinction!

What about extinction?

Here a small change can make a huge difference - extinction!

We can describe this mathematically, too:

population growth = population $\times$ (5 - population) $\times$ (population - 10) $\div$ 5

What to take away

Sometimes small things make a small difference (stable)

What to take away

Sometimes small things make a small difference (stable)

... and sometimes they make a huge difference (unstable)

What to take away

Sometimes small things make a small difference (stable)

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A small change can mean the difference between survival & extinction.

What to take away

Sometimes small things make a small difference (stable)

... and sometimes they make a huge difference (unstable)

A small change can mean the difference between survival & extinction.

Mathematics can predict when small changes matter.