Brown/Paris Numerical Analysis: Problem set 5

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1 Numerical solution of non-linear equations

Last week we solved a bunch of linear systems of equations. This week we're going to learn how to solve problems that are unsteady in time and to do that we'll need to be able to solve non-linear equations.

Goal: Numerically find the roots of the equation

$$x^2 + \sin x = 0 \tag{1}$$

- Start by making a plot and identifying what the function looks like, how many roots the equation has, and take a rough guess at what the roots are.
- Using your plot to bracket an estimate for the answer, use the bisection method to numerically calculate the root.
- Use Newton's method and the secant method to find the same roots.
- Check your results by making sure that the residual of the equation matches the tolerances used in the methods (try $\epsilon_{tol} = 10^{-5}$). Plot the convergence rate of these methods to see how the iterations scale with with the tolerance parameter.
- Newton's method works by approximating f with a linear function. Can you come up with your own method that does the same thing for a quadratic function? How does this change the rate of convergence.