CFD - Lecture 9

- Odds & ends,
  - quiz next week (informal, nearly direct from class notes)
  - final projects
    - groups are fine
    - make an appointment to discuss

In upcoming HW, include an email

My group is: (names)

My project title is: ( )

My project will study/produce: (deliverables)

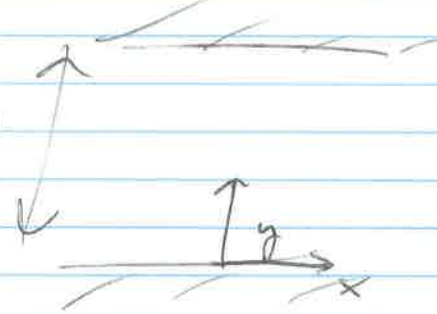
In my 8 min presentation, I will show ( )

Today
- Recall laminar channel flow
- What happens when Re ↗ ?
- Some videos
- An overview of turbulence modelling
- Turbulent flow over a sphere

Turbulence is an extremely rich topic, but to solve practical problems, need a rough understanding of how to get a decent ~~model~~ tractable model.

"When I meet god, I am going to ask two questions: why relativity and why turbulence? I really believe he will have an answer for the first" - Heisenberg

Pouseille flow



Assume ~~$u = f(x)$~~

$\vec{u} = \langle u(y), 0, 0 \rangle$

$p = p(x) = \text{const.}$

$\nabla \cdot \vec{u} = \partial_x u(y) = 0$ ✓

$$\cancel{\partial_t u} + u \cdot \nabla u = -\nabla p + \nu \nabla^2 u$$

$$u(0) = u(H) = 0$$

$$\cancel{u \partial_x u} + \cancel{v \partial_y u} = -\partial_x p + \nu \left(\cancel{\partial_{xx} u} + \partial_{yy} u \right)$$

$$\partial_{yy} u = \partial_x p$$

$$\nu \frac{d^2 u}{dy^2} = \frac{\partial p}{\partial x}$$

$$\nu \frac{du}{dy} + \cancel{c_1} = y \frac{\partial p}{\partial x} + C_1$$

$$\nu u(y) = \frac{1}{2} y^2 \frac{dp}{dx} + C_1 y + C_2$$

$$0 = u(0) = C_2$$

$$0 = u(H) = \frac{1}{2} H^2 \frac{dp}{dx} + C_1 H \rightarrow C_1 = -\frac{1}{2} H \frac{dp}{dx}$$

$$\nu u(y) = \frac{1}{2} \frac{dp}{dx} \left(y^2 - Hy \right)$$

$$u(y) = \frac{1}{2\nu} \frac{dp}{dx} y(y - H)$$

$$Q = \int_{\text{inlet}} u \cdot n \, dA = -\int_0^H u(y) \, dy \qquad = -\frac{H^3}{6}$$

$$= -\frac{1}{2\nu} \frac{dp}{dx} \int_0^H y(y - H) \, dy$$

$$\boxed{Q = \frac{H^3}{12\nu} \frac{dp}{dx}}$$

Statistical treatment of turbulence

- Seen from videos a wide range of length scales

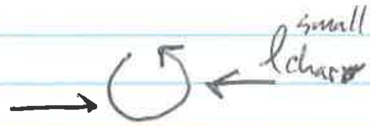
$$l_{char}^{big} \rightarrow \text{geometry dependent}$$

$$l_{char}^{small} = ?$$

~~To estimate scale of smallest eddies, consider the viscous length scale~~

- At some point viscous forces dominate

$$F \sim \nu \nabla u \sim \frac{\nu\, u_{char}^{small}}{l_{char}^{small}}$$



- Idea
  - turbulence is fundamentally unsteady
  - separate velocity into an average and fluctuating component

$$u = \bar{u} + u'$$

  - Molecular dynamics analogue: For molecular fluctuations, the bulk flow is governed by ensemble average

$$\bar{u}_{molecular} \sim \frac{1}{N} \sum u_i$$

explain ensemble average

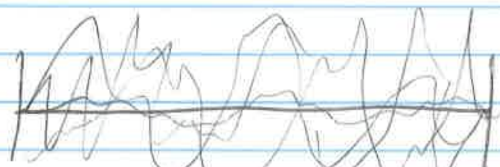
  - Ergodic hypothesis
    - can exchange ensemble average with time average

- So if we took a bunch of snapshots of the channel flow problem, and average them together, we obtain a mean profile $\bar{u}$,

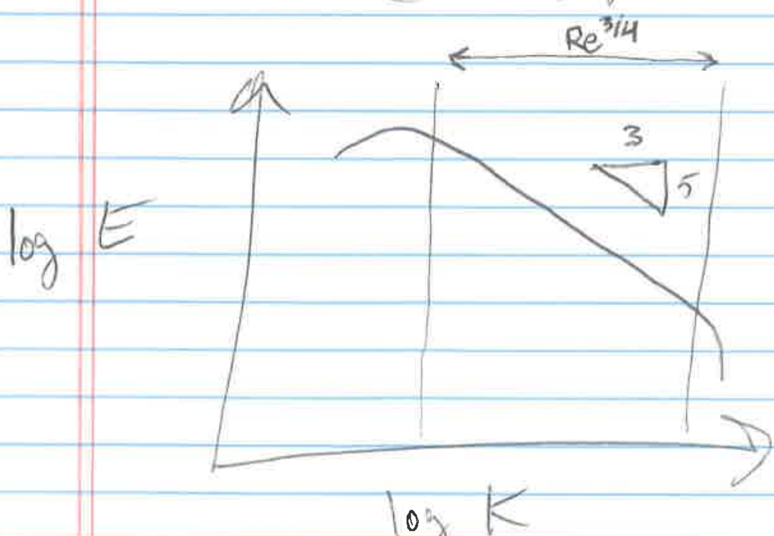
and obtain a collection of "random" variables $u' = u - \bar{u}$

- Why "random"?

- How does $\bar{u}$ compare to $u \sim y(H-y)$?

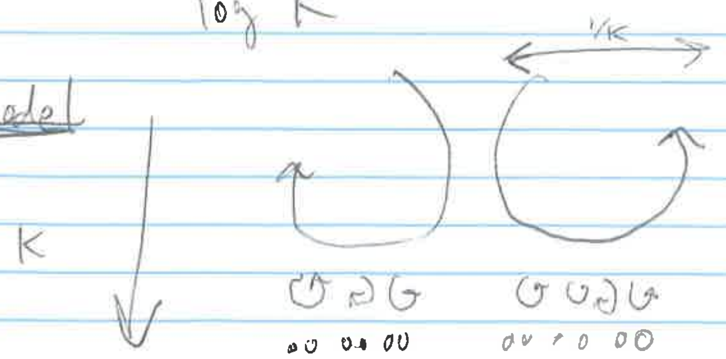


- Study the structure of these fluctuations by considering the energy contained in each mode in its Fourier spectrum



- $E \sim \| u' \|^2$
- $K \sim 1/\text{eddy size}$
- lengthscale ratio $Re^{3/4}$
  $\rightarrow$ in 3D we need $Re^{9/4}$ scales

Model



Re in nature	
Fish (small slow)	1
Blood flow	10-100
MLB pitch	2×10^5
Fish (fastest)	10^6
Whale	10^8
Big ship	10^9

- Need to model

DNS LES RANS

Goal: resolve $\bar{u}$, model u'

Define averaging operator as $\overline{u\bar{v}} = \bar{u}\bar{v}$

$\overline{(u')} = 0$ and assume commutes w/ derivative

Stick $u = \bar{u} + u'$ into N-S

$$\partial_t u + u \cdot \nabla u = -\nabla p + \nu \nabla^2 u$$

Mom. $\partial_t(\bar{u}+u') + (\bar{u}+u')\cdot\nabla(\bar{u}+u') = -\nabla \bar{p}+p' + \nu\nabla^2 \bar{u}+u'$

Mass. $\nabla\cdot(\bar{u}+u') = 0$

Take average of mass

$$0 = \overline{\nabla\cdot u + u'} = \nabla\cdot\bar{u}+\bar{u'} = \boxed{\nabla\cdot\bar{u} = 0}$$

For mom. Similarly all the linear terms' fluctuating part drops out, except

$$(\bar{u}+u')\cdot\nabla(\bar{u}+u') = \bar{u}\cdot\nabla\bar{u} + \bar{u}\cdot\nabla u' + u'\cdot\nabla\bar{u} + u'\cdot\nabla u'$$

Take average $= \overline{\bar{u}\cdot\nabla\bar{u}} + 0 + 0 + \overline{u'\cdot\nabla u'}$

$$= \bar{u}\cdot\nabla\bar{u} + \overline{u'\cdot\nabla u'}$$ ← Reynolds stresses

And we get Reynolds-Averaged N.S eqns (RANS)

$$\begin{cases} \partial_t \bar{u} + \bar{u}\cdot\nabla\bar{u} = -\nabla\bar{p} + \nu\nabla^2\bar{u} + \boxed{\overline{u'\cdot\nabla u'}} \\ \nabla\cdot\bar{u} = 0 \end{cases}$$

Same as N-S but with new term

Assignment: (due 5/1)

The drag force can be expressed as

$$F_d = \frac{1}{2} C_D \rho u_\infty^2 A$$

Drag coefficient, density, Frontal Cross-sectional area

1. Set up 3D sphere geometry from Holtzmann-CFD website
2. Confirm that you can match drag for low-Re case
3. Add K-ε turbulence model (we'll cover this next week) and reproduce the graph of C_D vs Re for

$$Re \in [10^0, 10^1, 10^2, 10^3, 10^4, 10^5, 10^6]$$