CFD - Final lecture

Logistics:

- Out of town Tues - Sat
- Everyone should have talked to me about their project
- Next week, project workshop with some tutorials
- If there is a topic you'd like covered in workshop, let me know
- Course evaluations

Today:

- Finish turbulence modeling
- Into to Foam source code
- Turb. modeling tutorial
- Intro to snappy Hex Mesh
- Final HW

Last time:

- Recall idea of turbulence models:

$$u = \underset{\substack{\uparrow \\ \text{mean}}}{\bar{u}} + \underset{\substack{\uparrow \\ \text{fluctuations}}}{u'}$$

- Many ways to generate a mean
  - ensemble averaging
  - time average + ergodic hypothesis

$$\bar{u} = \frac{1}{\tau} \int_{t-\tau}^{t} u(t)\, dt$$

- After introducing averaging, we obtained the RANS equations

$$\begin{cases} \partial_t \bar{u} + \nabla\cdot \bar{u}\bar{u} = -\nabla\bar{p} + \nu\nabla^2\bar{u} + \nabla\cdot\underbrace{\overline{u'u'}}_{\text{Reynolds Stresses}} \\ \nabla\cdot\bar{u} = \nabla\cdot u' = 0 \end{cases}$$

Reynolds Stresses $= \vec{R}$

- How to model?
  - the hope is that we can write $\vec{R} = f(\bar{u})$ so that we have a solvable system

- Eddy viscosity model
  - turbulence described as "nature's stirring spoon"
  - Motivated by this, take a guess that by analogy to

viscous shear stresses $\nabla\cdot\left(\nu\left(\nabla u + \nabla u^T\right)\right)$ for molecular mixing

$$\rightarrow \boxed{\nabla\cdot R = \nabla\cdot \nu_{turb}\left(\nabla\bar{u} + \nabla\bar{u}^T\right)}$$

- Note that

$$[\nu_{turb}] = \frac{L^2}{T}$$

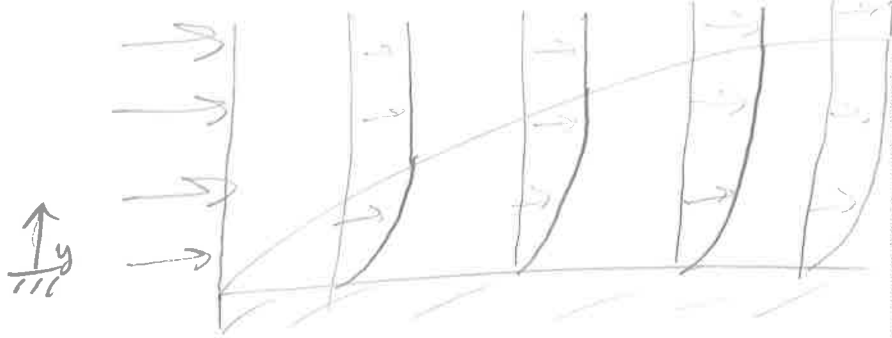
No rigor, lots of dimensional arguments

History

- zero - equation model (Prandtl, 1925)

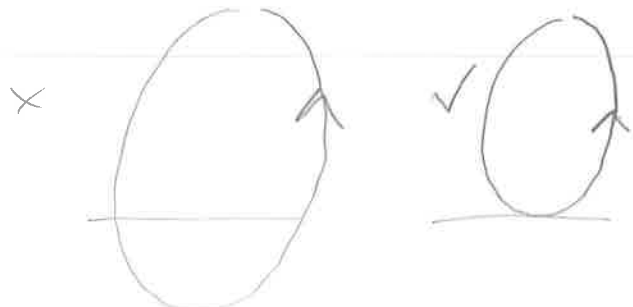
$$\nu_T = l_0^2 \left| \frac{\partial u}{\partial y} \right|$$

$$l_0 = \kappa y$$



- argument: ① right units
  ② size of eddy that can "fit" scales like distance to wall

- Good picture on wikipedia for Mixing length model



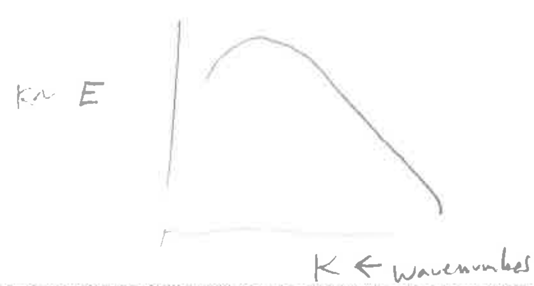
- one - equation model

Define turbulent kinetic energy

$$K = \frac{1}{2} \overline{\| u' \|^2} \qquad |K| = \frac{L^2}{T^2}$$

We'll see that there is a transport equation for K that can be derived similar to the RANS eqns

- Gives the overall amount of energy, but no idea about distribution over scales



$$\nu_t = K^{1/2} l_0$$

- where l_0 is a length scale parameter
- tune l_0 w/ experiments → limited range of application

two-equation model ← (accepted as the "right level of complexity")

Turbulent dissipation

- small eddies dissipate energy faster than large eddies
- we can cook up a term describing the rate that the fluctuations use up energy and get a timescale

$$\boxed{\varepsilon = \nu \, \overline{\nabla u' : \nabla u'}}$$

$$[\varepsilon] = \frac{L^2}{T^3}$$

K-ε model ← we'll do this in OpenFoam today

$$\boxed{\nu_t = C_\mu \frac{K^2}{\varepsilon}}$$

$C_\mu \approx 0.09$ ← experimentally tuned

So our RANS system will look like

$$\begin{cases} \partial_t \bar{u} + \nabla \cdot \bar{u}\bar{u} = -\nabla \bar{p} + (\nu + \nu_t) \nabla^2 \bar{u} \\ \nu_t = C_\mu \dfrac{k^2}{\varepsilon} \\ \text{transport eqns for } k, \varepsilon \end{cases}$$

we'll take a look at k eqn

<u>exact eqn</u>

$$\frac{\partial k}{\partial t} + \bar{u} \cdot \nabla k = \vec{R}_{ij} : \nabla \bar{u} - \varepsilon + \nabla \cdot \left(\nu \nabla k - \frac{1}{2} \overline{u'u'u'} - \frac{1}{\rho} \overline{p'u'} \right)$$

$$R_{ij} = -\overline{u'u'}$$

- <u>High order closures</u> same situation as RANS eqns (the <u>closure</u> <u>problem</u>)

Assume form

$$\partial_t k + \bar{u} \cdot \nabla k = \underbrace{\nabla \cdot D_k \nabla k}_{\text{diffusion}} + \text{production} - \text{dissipation}$$

exact

$$= \nabla \cdot ((\nu + C_3 \nu_T) \nabla k) - R_{ij} : \nabla \bar{u} - \varepsilon$$

<u>idea:</u> postulate something computable with some experimentally tweakable parameters

Source Code

In todays tutorial, we'll set up a turbulence model in our nsFoam solver and compute the drag over a sphere using our completed solver & snappyHexMesh.