GEORGE MACKEY


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GEORGE WHITELAW MACKEY was born on 1 February 1916 in St. Louis, Missouri, and grew up in Texas. He received his B.A. in physics from Rice University in 1938 and his Ph.D. in mathematics from Harvard University in 1942. His thesis, entitled "The Subspaces of the Conjugate of an Abstract Linear Space," was written under the direction of Professor Marshall Stone. After a year teaching at the Illinois Institute of Technology, he returned to Harvard, where he spent the entire rest of his career. He became full professor there in 1956 and had twenty-three Ph.D. students before he retired in 1985. He died on 15 March 2006, soon after his ninetieth birthday.

Mackey's scientific work concerned the three areas of unitary representations of locally compact groups, ergodic theory, and the foundations of quantum mechanics. Although his specific theorems may be technical, he was a generalist at heart and he pursued a consistent vision throughout his career. This vision was that there is an extraordinary unity in the deep mathematical threads that bind together all of analysis, group theory, mathematical physics, and even number theory. He expressed his grand vision in two books, based on lectures given at Chicago and Oxford. These books have inspired a whole generation that went on to further development of his ideas. The idea that mathematics is a single grand edifice with its internal logic is an old one, dating perhaps back to Euclid. In the twentieth century, the French collective known as Bourbaki created an unflinchingly abstract and general, some might say even inhuman, embodiment of this vision. Mackey, on the other hand, built his edifice largely on group theory, the idea that the underlying symmetries are the core of all deep theorems and mathematical structures. His unification was rooted in concrete results that intertwined in startling ways.

One of his first results foreshadowed many of the themes he developed later in his career. This result concerned the famous Heisenberg Commutation Relations, the simplest expression of the uncertainty of the quantum world. Heisenberg theory stated that the operators' measuring position and momentum do not commute, and, in fact, their commutator is a multiple of the identity, and thus a particle cannot have both precise position and precise momentum in any state of the world. Mackey studied variants of this statement in which the Euclidean group of positions and momenta was replaced with an arbitrary locally compact abelian group A and its dual. His theorem stated that there is an essentially *unique* set of operators associated to A and its dual that skew-commuted like the Heisenberg operators.

At about the same time, Wigner realized that the elementary particles of physics are associated to representations of the groups of symmetries of space and space-time and took the first step toward classifying these. Potential applications to physics were never far from Mackey's mind, and Wigner's work was, I believe, one of the strong motivations for Mackey when he began to consider systematically the unitary representations of all locally compact groups. The first major piece of work for which Mackey is famous is his analysis of the representations of *non-simple* locally compact groups, that is, groups with normal subgroups, and the creation of what many now call "the Mackey Machine." This machine is a program for classifying the representations of a group G via the process of inducing representations from those of its subgroups. He showed how in a certain set of "good" non-simple cases, all the representations of G can be constructed from those of its normal subgroup N and of quotient groups H/N, H a subgroup of G containing N. As was true of many of Mackey's results, he took ideas from the simple case of finite groups and finite-dimensional representations and had to work hard to extend them to continuous groups and infinite-dimensional Hilbert spaces.

While developing this theory further, the difficult cases of his theory led him directly to questions in ergodic theory. This is the theory of group actions like rotating a circle through an irrational angle, which act as though they are transitive because one cannot divide the circle into two parts of positive length each preserved under the rotation. This led Mackey to his startling and fundamental generalization of ordinary groups, which he called *virtual groups*. If a group G acts transitively on a space X, then X can be considered as a quotient of G by an ordinary subgroup H; Mackey said, if the action is not transitive but is merely ergodic, we should still think of X as a sort of quotient, but now by a virtual subgroup. In various ways, his work here was an important precursor to the theory of Connes, known as *non-commutative geometry*. Throughout his career, he continued to relate his work to all the developing ideas in quantum field theory and elementary particle physics.

Even though my research has been in different areas of mathematics, George was one of the biggest influences on my own mathematical career and a very close friend. It was George who showed me for the first time the beauty of the world of mathematics. I met him as a sophomore at Harvard in 1954 and read his wonderful lecture notes. In those days, he led the life of an English don, living in a small apartment with one armchair and a stereo. This was my first exposure to what higher mathematics is all about and to the way someone can be so devoted to his vision, something I had never seen until then.

George Mackey's great influence on the mathematical scene was based not only on his deep and beautiful work, but on his unique personality and his unwavering insistence on thinking through every issue, political and social as well as scientific, from his own logical perspective. Back in the sixties, government funding of mathematical research was just starting, so, of course, everyone was applying. Not George: he rocked the Boston mathematical community—not for the last time—by saying what no one else dared: the government was wasting its money because all of us would do math all year without the 2/9th raise they were offering. *He would not take it.* Besides, in a darker note, he predicted all too accurately that when we were bought, the government would try to influence our research. To varying degrees in different fields, this has come to pass. As an applied mathematician for the last twenty years, I find it downright embarrassing to see how much government pressure is being applied to create interdisciplinary collaborations. If they happen, great. But this should be each mathematician's personal choice, governed by his interests.

George's outspokenness, his brutal honesty, probably got under everyone's skin at some point. He never adjusted his message to his listener. But he often articulated thoughts that we shied away from. Certainly, his carrying his clipboard and catching an hour to do math alone while his wife and daughter went to a museum is a fantasy many mathematicians harbor. George maintained his intellectual schedule through thick and thin. Perhaps my favorite memory of one of his unorthodox points of view is this. I asked him once how he survived his three years of Harvard's relentlessly rotating chairmanship. His reply: he was most proud that, under his watch, nothing had changed; he left everything just as he had found it. A true conservative, for him the right values never changed.

As I said, we were close friends for all his life. In fact, we continued to meet for lunch, George's favorite way of keeping in touch, until his deteriorating health overtook him and he was forced to retreat to a nursing home. He would always walk from his house on Coolidge Hill to the faculty club, which was perhaps his chief source of physical exercise. Over lunch, we would first go over whatever kind of math each of us was playing with at that time. But then we also talked philosophy and history, both of which attracted George a great deal. He liked the metaphor that perhaps our essential being is not that of physical bodies in what is called the "real world." Rather, as in the movie *The Matrix,* our true being might be elsewhere, but there we are encased in a sort of diving suit that reproduces the sensations and transmits motions to what we falsely believe is our body.

George was not religious in the conventional sense. But I think his proof of the existence of God was the intertwining of all branches of mathematics and physics. Many mathematicians have been frustrated by the seeming intractability of the problem of reducing quantum field theory to precise mathematics. But here George was the perennial optimist: for the whole of life, he remained sure that the ultimate synthesis was around the corner, that there would be a supremely beautiful and rigorous mathematical theory that would underpin particle physics, and that group representations would play a central role.

As I learned many years ago from reading about them in his notes, intertwining operators were one of George's favorite mathematical constructs. But I think they are a metaphor for much more in George's life. His wife and daughter were his wonderful support system and they intertwined George with the real world. There were many wonderful gatherings at their house. George and Alice maintained the long tradition of proper and gracious dinner parties for the mathematical community in Cambridge, long after it had gone out of fashion for the younger generations. They were truly his lifeline, the hose bringing air to that diving suit.

Elected 1971

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