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Mathematics Belongs in a Liberal Education

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ABSTRACT

In higher education today, mathematics has been marginalized: except to a tiny elite, it is either taught as a tool required for the study of other sciences or it is entirely absent. Yet mathematics has been and is an essential ingredient in our understanding and mastery of the physical world, our economic life, our information technology and, increasingly, the workings of biology. Its key insights belong in the toolkit of all educated people. Unfortunately, present-day school instruction isolates mathematics from the tangible things that were its historical roots: the geometry and physics of the world, data and commerce, chance and randomness. We will look at the history of mathematics in western culture, the nature of the present-day alienation of our society from mathematics and suggest some ideas for bridging this gap.

KEYWORDS *higher education, liberal education, mathematics education*

INTRODUCTION

THE CLAIM IN my title may seem uncontroversial: what teacher would deny that every educated person should know some mathematics? In fact, it is radical. Mathematics is not part of what we call a liberal education today, and to make it so will require a significant revision of our thinking as teachers. Every educated person, I argue, should know far more mathematics than he or she knows today, and in fact should learn how mathematics is an integral part of our culture. I fear that my thesis will be equally distasteful to humanists and my fellow mathematicians alike. In the past half century, mathematics has had virtually no role at all in the curriculum of a liberal education. Presumably, the humanists who molded these curricula felt that mathematics was

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merely a tool needed for technology and was not truly intertwined with the great themes of our culture. I want to argue the opposite – that mathematics has been and is linked in fundamental ways with the development of our culture and our thinking, and that its presence can deepen and contribute a hard edge to the softer humanist material of liberal education. On the other hand, I want to argue that teaching mathematical skills as usual, with a forced march through calculus, linear algebra and differential equations, is an equally large mistake if we seek to impart understanding of what mathematics is and how it affects our lives. We need, in fact, to adapt the approach to teaching mathematics so that it is not a test of endurance nor an extended IQ test, but is exciting and clearly relevant to understanding our world.

HOW MATHEMATICS HELPS YOU UNDERSTAND THE WORLD

In the first section of this article, I want to argue for my case that mathematics is indeed an important ingredient of our understanding of the world. Mathematics played a central role in the liberal education taught for nearly two millennia. From Plato through medieval times, the core of higher learning was the *quadrivium*, which consisted of the four subjects arithmetic, geometry, music and astronomy. This division of knowledge is traditionally attributed to the Pythagoreans and is described in Plato's *Republic*. It was preceded by the three basic subjects of elementary education, the *trivium*, made up of grammar, rhetoric and logic. Together they made up the seven 'liberal arts'. The idea of structuring the education of learned men around these seven liberal arts spread from the ancient world through learned Muslim communities to medieval Europe.

Several key ways of understanding the world with the help of mathematics were developed in the *quadrivium*. Astronomy, of course, has preoccupied thinkers in every culture. The central challenge of astronomy arises from the fact that the sun, stars, moon and planets move in nearly periodic yet subtly variable ways. Measuring, recording and modeling these motions, not to mention seeking their *meaning,* have tested the skills of each civilization. Serious arithmetic and geometry (e.g. Ptolemy's use of tables of chords – essentially sines and cosines) was called for to express humanity's increasing knowledge of these motions. In another direction, one can argue that the first stimulus to deeper mathematics was Pythagoras' theorem (which was, in fact, known to the Babylonians and the Chinese as well as the Greeks). This theorem is extremely significant because it lays out the basic link between the multi-dimensional geometry of our world and arithmetic, and shows how distances between multiple points are all related to each other. With

Pythagoras' theorem in hand, we can lay out the world with string. A third discovery that tied the subjects of the *quadrivium* together was that integral ratios between the lengths of strings produce harmonious chords. This was another deep link between our perceptions and arithmetic. All of these discoveries were seen by thinkers of the ancient and medieval world as fundamental to their understanding of the world and to their philosophies.

These examples of mathematics in the ancient world may have lost some of their fascination and seem merely esoteric in our age of high technology. But a second thread of ideas wove developing mathematics into the material culture of the world: the development of accounting and its prerequisites, fast algorithms for the four operations of arithmetic and efficient notation for numbers. Recall that serious thinkers such as the many sided Renaissance genius Luca Pacioli developed and wrote about the new science of accounting. In fact, Alfred Crosby's book, *The Measure of Reality* (1997), argues that the trigger for the rise of European civilization starting in the Renaissance was the rediscovery of the efficacy of accurate measurements. He finds this in three areas: in the construction of town clocks to regulate the city dweller's life; in the drawing of accurate maps as explorers voyaged further; and in double-entry book-keeping, which allowed complex commercial transactions.

Important though these examples are, I would argue that the definitive case for the centrality of mathematics in our understanding of the physical world comes from the work of Isaac Newton. When describing the highpoint of his work, it is customary to focus on the synthesis given by his Law of Gravity, traced perhaps facetiously by Newton himself to his being hit on the head by a falling apple. His idea was that one and the same force pulled the apple to earth and pulled the moon to the earth. But as the moon is moving so fast, its fall towards the earth is just enough to restrain it from flying off into space and keep it in a circular orbit. It is a small mathematical calculation, which can be done on the back of an envelope, to check that both forces can have the same origin if you allow the strength of gravity to decrease by the square of your distance from the attractive body, the earth.

From the point of view of where Newton fits in a liberal education, however, I would instead underscore the sweeping perspective introduced by his law of force, the equation $F = ma$. In English, the law reads 'force equals mass times acceleration'. Galileo had said 50 years earlier:

> Philosophy is written in that grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language . . . in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles and other geometric figures. (*Il Saggiatore* [*The Assayer*], 1623)

Note that Galileo was still under Euclid's influence and saw triangles and circles as the essential mathematical ingredients of the physical world. Newton's force law finally displaced Euclid from center stage, instead asserting that *rates of change* were the things to look at. Euclid had written about an essentially static universe and, though there were some earlier attempts, it was Newton who taught the world how to make precise descriptions of a dynamic universe. He did this using the mathematics of rates of change, by inventing the techniques which are called calculus. Acceleration is simply the rate of change of velocity and velocity is simply the rate of change of the position of an object. Equations involving rates of change are called *differential equations* and his law of force was the first such equation. I think it is quite correct to update Galileo's pronouncement and say: *differential equations were the language in which God wrote the laws of the physical universe*.

It is not difficult to put this insight into simple terms. Science in general seeks to be able to predict the future in as many situations as possible. In essentially all situations where such predictions are possible, the gold standard is first to find a set of numbers of that express part of the state of the world at an instant of time. To make this concrete, if one is dealing with the planets, these numbers are the positions and velocities of each planet at the given instant. Or if one is dealing with an ocean wave, these numbers are the positions and velocities of each drop of water in the wave. (Both these examples were considered by Newton.) These numbers are called the *state variables*: they are merely the measurements you can make to describe some part of the world. Then a differential equation is a formula giving the rates of change of these numbers in terms of their present values. The rate of change of velocity is acceleration, so you see why the original law of force is just such an equation.

By many methods, some going back to Newton, such formulas can usually be solved uniquely to predict the state variables in the future in terms of their values in the present. This is a sweeping and pervasive approach. It has been extended by biologists to model the workings of a cell and by economists (not 100 per cent successfully!) to track the fluctuations in the business world. It remains the dominant paradigm in science and technology today. It is a perspective which every educated man or woman should absorb. Once this idea is understood, one can easily extrapolate and have quite a clear idea of what most of the supercomputers in the world today are grinding away at – they are just solving differential equations.

What other basic mathematical facts should be in the toolkit of the educated layperson today? Buried in the middle of high school math drill, our children encounter graphs. Most every parent measures their children's height each year and marks it on the wall: plotted against their age, you have a very informative graph of their development. Graphs are a hugely undervalued tool in our understanding of the world. Graphing data was invented by the scholastic philosopher Nicole Oresme in the 14th century. He called it the theory of 'configurations' and, in fact, also invented so-called 'Cartesian' coordinates in order to do it. He imagined graphing not only physical quantities like speed as a function of time, or temperature as a function of space, but pain and even grace as functions of time too. His principles of accurate graphing have been promulgated very effectively by the Yale graphical artist Edward Tufte. Properly done, they are a huge help in visualizing data of all kinds. Sadly, newspapers and magazines, not to mention Presidential debates, either avoid graphing or make grossly inaccurate cartoons out of graphs (this is the sin Tufte inveighs against).

Intellectually, there is a profound leap here. In Euclid, configurations were sets of lines and circles, the natural inhabitants of fixed and absolute space. Oresme, on the other hand, had the inspiration to use space in an analogical way, creating images within it of quantities which lived elsewhere. The importance of graphs was first pointed out to me by Andrew Gleason at the time that he was asked how to meld mathematics into the revised Harvard Liberal Education, the 'core curriculum'. Unfortunately, perhaps because he did not bring in Oresme and graphing's distinguished pedigree, his idea was demoted into part of a mini-exam, called the 'quantitative reasoning requirement'.

My final example of a unifying mathematical idea is *p-value*. This is a number which is a probability. The whole idea of measuring the likelihoods of one or another event is a relative newcomer to the mathematical universe, going back only to the Renaissance mathematician Girolamo Cardano. Cardano was addicted to gambling and was driven to quantify his passion. Philosophers have been arguing ever since about what the numbers expressing probabilities really mean. What does it mean, for example, to say there is 50 per cent chance of your next child being a boy? This birth is a unique event that will never happen again, and, unlike flipping a coin, you cannot have a next child 1000 times in order to see how often the child is a boy. Putting that to the side, however, again and again in everyday life, we need to assess the strength of some evidence: when is a smudgy incomplete fingerprint reliable evidence of who committed a crime? When is a medical study good enough reason to cease taking the pill?

The gold standard here is the idea of *p-value*: what is the probability that the evidence could have arisen by chance if your conclusion is false. In the case of fingerprints, the p-value is the probability that the smudgy fingerprint resembles that of the suspect by chance, given the random variation in fingerprint characteristics. Strikingly a whole article about fingerprints appeared in the New Yorker without mentioning p-values (or any numbers for that matter). If people know a little bit about p-values, they will be able to read medical journals and form their own judgments about the latest drug tests. Aside from using Latin words for parts of our bodies, the articles are not so complicated.

Statistics is an important and intellectually challenging subject with some subtle pitfalls. One of these is the possibility of measurements being correlated without a causation relationship. I can illustrate this issue with a curious example from a class I taught. We had data on annual murder rates in various towns and also of the growth of the GNP. I asked the class to see if there was a correlation. You can make two graphs, one showing the number of murders, year by year, and another showing the GNP growth, year by year. To my surprise, it turned out that the graphs were somewhat similar, though the similarity was strongest if you compared the murder rate in each year with the GNP growth in the *next* year! This similarity is formalized by a formula for what is called correlation. Does this mean that the murders caused the economy to nosedive? This is highly unlikely. I toyed with the idea that hidden factors preceding the downturn of the economy were causing social stress before they appeared in the government's figures, and that this was causing more murders. Or maybe it was just an accident of the data. This example is just a curiosity but data of this type are being used all the time and are being given interpretations by so-called experts. People need to have some critical skills to judge such conclusions and know when alternative interpretations are reasonable. Statistics is a subject that should be known to every well-informed citizen.

WHY DO SO MANY PEOPLE HATE MATHEMATICS?

I now want to turn to a different side of my argument. With all of this obvious usefulness and impeccable pedigree, why has not mathematics played a central role in liberal education in the 20th and 21st centuries? A quite extraordinary thing has happened to our culture. I quote from Hans Magnus Enzensberger's beautiful essay *Drawbridge Up, Mathematics – A Cultural Anathema* which starts with the caricature:

> 'Stop, for heaven's sake! I hate math', 'Pure torture from the start of school. It's a total mystery how I ever managed to graduate' . . . Complaints such as these are heard all the time. Thoroughly sensible, educated people express them routinely with a remarkable blend of defiance and pride. (1999: 9)

He goes on to contrast this with the fact that no educated person could admit without shame that they dislike reading in this way! Why do we give a pass to those who hate math?

One source of this may be a deep-seated ambivalence towards calculation. Does memorizing your multiplication tables and learning strenuous algorithms like long division cause in some a permanent antipathy towards numbers? If so, what a shame! Learning arithmetic should be like learning to drive: an essential skill in our quantitative society that allows you to manage a lemonade stand or hoodwink your shareholders in a Fortune 500 company. What is particularly ironic is that now we have pocket calculators so there is no need for tedious division: you can check your miles per gallon by dividing your odometer reading by the gas pump reading without a pen or pencil. It is a case of the horse and buggy being replaced by the racecar: the pocket calculator can actually do millions of operations per second. It may be instructive to understand the long division algorithm but you certainly don't need to belabor it. It's a shame if you are averse to simple divisions because they are often very instructive, especially because they help you tame big numbers. For example, the cost of the Iraq war (as of November 2004) is approaching $1000 for each man, woman and child in the USA: you can do the division yourself and see if I'm lying. (My favorite SAT question is to estimate the speed at which your hair grows in *miles per hour*.)

What is even more exciting is the spreadsheet, for example Excel. Excel is the SUV of calculators: it is powered by a huge amount of arithmetic horsepower and it should let people enjoy discovering what their numbers can tell them rather than sweating over their arithmetic. Many, if not most, of the mathematicians of the past loved to calculate as well as to prove theorems and they would have drooled to have a tool like Excel. In fact, going back to Newton and his models of the world using differential equations, the vibrating string differential equation was one of the central examples studied in the 18th century. This is the equation which allows you to predict the motion of a taut string (like a guitar string), given its initial position and its initial velocity. It links Newton's theory with music, and all the major mathematicians of that century had a hand in analyzing it. It is a fact that you can numerically approximate solutions of the vibrating string equation in Excel with literally half a dozen key strokes! Having such technological tools changes the educational landscape. If used properly, these tools make mathematics (i) fun, not tedious, (ii) tangible because it's all there in front of you and (iii) visual because you can also graph all the quantities.

Of course, there are other reasons people hate math. I suppose the main one is that it is too abstract and it loses touch with anything the student can relate to his/her immediate world. When algebra is introduced, its raison d'être is seldom explained. Instead it is a game of symbols with weird tasks like factoring being required. Again I believe progress can be found in the use of computers. Programmers rarely use x and y because they are so unmemorable.

Their numbers are given names which are usually acronyms for their roles, for instance, they might use ACC1 for *a*, the acceleration, and MASS1 for *m,* the mass, of object 1. To describe a lemonade stand's finances by PRFT = RCPT – COST; RCPT = PRC 3 SALES, and so on is to use algebra to express the general principles of retailing.

Ironically, mathematicians have fostered this tilt to abstraction. Their love affair with abstraction can be traced to Euclid. Its most influential modern exponent is the infamous French mathematical cooperative, writing under the pseudonym Bourbaki. There is a further irony here. Mathematicians have a clear and consistent idea of what is beautiful in mathematics and use the term freely. For them, the beauty of mathematics is very similar to the beauty one finds in abstract art or architecture, or in music. Yet these latter disciplines have the advantage of producing works which are directly perceived by the senses and therefore are called beautiful by the public at large. The mathematicians feel cheated: their beautiful toys receive no praise or applause except by their tiny community of peers. But the people who are really cheated are the non-mathematicians who have not been taught that there is beauty in mathematical abstraction too.

Enzensberger raises an interesting point in the essay quoted above. Perhaps, he suggests, for most people the love of mathematical abstraction is lost rather than never learned, perhaps it is always there at an early age but is not adequately nurtured. Take Xeno's paradox about Achilles not being able to catch the tortoise. The tortoise starts out 100 feet ahead of Achilles, but when Achilles has gone 100 feet the tortoise has moved one foot further. Every time Achilles catches up to the last position of the tortoise, the tortoise has moved on a little more and so an infinite number of catch-ups are needed. Xeno asked how can the infinite number of required steps be taken in a finite amount of time? Almost every child who hears this riddle is fascinated by it. It is a joke because it does, and yet also does not, make sense, and kids enjoy this.

Here is another example of an appealing problem which leads directly to quite abstract ideas: you are given a car hitched to a trailer and you are asked how you turn the wheel and move the car so that, after some moves, you back the combination precisely into a parking space. This is a puzzle that will easily engage a class for an hour or more. To make it harder, you can hitch a second trailer behind the first. But it is also not hard to use this concrete problem as the foundation on which to build the abstraction of non-commutative groups of operations. Here commutative describes steps, call them A and B, which can be taken in either order with the same result; non-commutative means that if you do A first, B second, the end result is different from doing B first, then A.

In fact, when children master language, they are acquiring thousands of very abstract concepts at a rapid pace. How does a four-year-old come to understand the meaning of the word 'loyal'? Numbers themselves are a very abstract concept that every child picks up. So why should it be hard to acquire more abstract mathematical concepts? Since people easily learn to love abstract art, we should be able to grow their understanding of the abstract in math rather than suppressing it.

BRINGING MATHEMATICS ALIVE

I want to conclude with a few ideas for improving mathematical education. At the present time, mathematics, except in a few experimental curricula, is taught in isolation, as a subject with no connections with the rest of our culture. It starts with the forced march through the four arithmetic operations, then on to fractions and real numbers, to algebra, and simple bits of Euclidean geometry in high school, to calculus bridging high school and college, to advanced calculus and abstract algebra and so on. The entire establishment, from grade school home room teachers through research professors at universities, has a stake in this system, which they know well and teach each year. But where are the links with physics in this system, the links with accounting and its big brother, economics? If there is statistics, are real data used or, even better, are they gathered by the class? Sometimes you hear that you need the math first before doing the physics or that, if some students have done the physics, many haven't, so don't use examples from physics. These are excuses: mathematics did not grow in isolation even though some mathematicians prefer it that way.

This separation on a professional level of pure from applied mathematics, and of applied mathematics from physics and engineering, is a recent mistake. Virtually every mathematician before 1900 did both pure and applied research. Many continued to do so until about 1950 when the separation became institutionalized and the two communities went their separate ways. The idea of pure mathematics governed by its internal rules and developed by pure logic was another side of the same cultural movement which brought us impressionists and cubists, Bauhaus architecture and 12-tone music. The modernist movement manifested itself in many fields and mathematicians were influenced by and influenced in turn artists, architects and musicians. This movement is now spent and we need to reverse its excesses: do not isolate mathematics, but teach it as an integral part of the world – measuring speed, height, weight; commerce (costs, sales and profit) and gathering data. My proposal is that in college, introductory mathematics should be taught as an integral part of history, science and technology. St John's College in

Maryland does the first, going to the extreme of teaching calculus through reading Newton and Leibniz (a tough assignment).

I want to give three specific examples. The first concerns the treatment of Pythagoras' theorem. Instead of simply proving it, the class can check it by measuring triangles on the earth. Just because it is called a 'theorem', this does not mean it is necessarily true! All applications of theorems to the world are based on the assumption that certain models for something hold in the world. Pythagoras' theorem depends on the parallel postulate which, since the work of Einstein, is known to be only approximately true in the real world. Kant thought the axioms of Euclid belonged in his category of the *synthetic a priori,* but notwithstanding this, Gauss went out in the early 19th century and measured the angles of a triangle formed by three peaks in Germany. Confirming the approximate validity of Euclid's axioms, their sum did indeed come out to be 180 degrees to within the accuracy of his instruments. Of course, he had an idea of why it might not have come out that way, as he had been developing the theory of non-Euclidean geometry. Although Euclid's geometric model was confirmed by Gauss to the limit of accuracy of his instruments, Einstein discovered a century later a deeper geometric model and a more accurate truth: that, due to gravity, the sum of these angles is a very tiny bit greater than 180. The whole exercise is a living example of how theory, models and experiment interact over time. As an aside, measuring the size of the earth is another good example of an experiment with your classroom geometry: one can do it in the same way as Eratosthenes or, if you live near the sea or a large lake, one can measure how objects dip below the horizon at a distance of several miles.

The second example takes up the third element of the *quadrivium*: music. Arguably the most beautiful and simplest idea in the mathematics after calculus is something called Fourier analysis. Going back to Newton, it was known that there was one simple law which describes all simple oscillations – for example, the movement of a weight bobbing on a spring or of a pendulum with small motions. It is called a 'sine wave'. Playing any music, the air pressure oscillates in time, repeating itself several hundred times a second. One can record exactly how the air pressure changes and graph it. For some instruments, such as a silver flute at high frequency, it turns out to be very close to sine wave. But usually it is more complex and this complexity gives the instrument its characteristic tone. The idea of Fourier analysis is that the more complex sounds can be reproduced exactly by playing at once pure sine wave tones at all the 'harmonic' frequencies that are at the original pitch, at double this pitch, triple, and so on. The air pressure waves add together and produce a more complex oscillation. Abstractly, it says that any periodic function is a sum or superposition of pure sine waves of varying strengths and phases at all harmonic frequencies. The idea has an extraordinary history, with some of the greatest mathematicians of the 18th century denying that this could be done until the polymath Fourier picked it up in the 19th century. And it then gave rise to an explosion of technology, in particular the possibility of sending zillions of radio, TV, cell phone signals, all simultaneously through the 'ether'. The theory, if one ignores the nitpicking details, is also extraordinarily elegant and simple. Sadly, the topic is taught today fragmented between four or five courses and never at an accessible level. In fact, it can be taught to Freshmen and worked out in Excel and it can be verified using recordings of music of any kind.

My last example was inspired by Peter Galison's excellent book *Image and Logic* (1997). It is the design of the atomic and hydrogen bombs. Galison's book has a chapter on the story, intertwined with an introduction to the most startling advance in numerical mathematics of the last 50 years: the Monte Carlo method. The amazing and counter-intuitive idea is that throwing dice can lead to extremely fast ways of computing some things which are otherwise totally out of reach, that is, things which, if calculated in more straightforward ways, take orders of magnitude more time. In the 1940s and 1950s, three brilliant mathematicians were working on the design of both bombs at Los Alamos: Nicolas Metropolis, Stan Ulam and John von Neumann. They were, of course, trying to follow the course of an explosion, when huge numbers of particles and atoms (e.g. the number is around 10 with 25 zeros after it) were colliding and flying around at increasing speeds. The goal was to see if the bomb stayed together for the split second needed for it to explode. Their idea was very simple: do not try to keep track of every particle but instead follow a tiny sample of only 100 particles and assume that the probability of these particles hitting other particles is given by odds depending on what is around them and how dense it is. Throw a coin (literally, use a 'random number generator') to decide randomly, while the computer is running, when and where the next collisions will take place. Throw a coin again to decide if an atom will split, and keep tracking with this guesswork the effects of these particles and their new speeds as time goes on. You create a pollster's universe, a possible scenario for this small part of reality. But it turns out that, unless you are very unlucky, you do track more or less correctly what the whole bomb is doing. This is actually how the hydrogen bomb was designed, using the biggest computers available in the early 1950s. But, now, computers are so much more powerful that an undergraduate can repeat much of this calculation using a PC and Excel. It is a great surprise to most students that throwing dice is so useful.

C.P. Snow, in his 1959 Rede Lectures, proposed that our society has split into 'two cultures'. This was not so until recently and it would be sad if this schism were permanent. My belief is that if math is taught more loosely, in everyday language, with examples, history and numerics, it will be quite accessible and can resume its rightful place in the education of the next generation. As I said at the beginning of this article, I realize that some humanists will not welcome mathematics into a liberal education curriculum for the good reason that they remember it as a mechanical and meaningless part of their own education. And equally, the idea of putting mathematics in everyday language will be decried as 'dumbing down' their subject by many of my colleagues. As Ian Stewart put it in the Preface to his book *The Problems of Mathematics*, it is like asking them 'to lie a bit', because, for them, the only exact and clear way to express a mathematical truth is to replace every fuzzy English word with a technical term and use the most abstract and general setting as possible. All this notwithstanding, it is obviously better to try to repair this split. This article is intended to encourage professors and teachers to try more experiments to convey the excitement of mathematics to a larger subset of educated people and to give mathematics its rightful place as part of a true liberal education.

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