AM 41: Mathematical methods in the brain sciences

Class Outline *(last updated November 18, 2004)*

0. **Introduction**

- 0.1 Three Main Topics
  - 0.1.1 Differential Equations
  - 0.1.2 Probability and Statistics
  - 0.1.3 Information Theory
- 0.2 MATLAB
  - 0.2.1 Plotting sinusoids
  - 0.2.2 Computing Fibonacci numbers
  - 0.2.3 A simple differential equation
  - 0.2.4 Numerical integration

1. **Differential Equations**

- 1.1 One-dimensional Differential Equations
  - 1.1.1 Differential equations and their numerical integration
  - 1.1.2 Linear equations
  - 1.1.3 Some non-linear equations
  - 1.1.4 Qualitative analysis
- 1.2 Two-dimensional Differential Equations
  - 1.2.1 Examples
  - 1.2.2 Qualitative analysis
    - 1.2.2.1 The Phase Plane
  - 1.2.3 Pairs of linear equations
    - 1.2.3.1 Stability analysis
    - 1.2.3.2 Nullclines
  - 1.2.4 Pairs of nonlinear equations
    - 1.2.4.1 Stability and local analysis
    - 1.2.4.2 Example: two neurons
    - 1.2.4.3 Linear approximations near equilibria
    - 1.2.4.4 Limit cycles and the Poincaré-Bendixson Theorem
    - 1.2.4.5 Example: a two-neuron oscillator

2. **Probability and Statistics**

- 2.1 Probabilities and Random Variables
  - 2.1.1 Probability spaces
    - 2.1.1.1 Sample spaces
    - 2.1.1.2 Events
    - 2.1.1.3 Probabilities
    - 2.1.1.4 Conditional probabilities
  - 2.1.2 Random variables
    - 2.1.2.1 Definition
    - 2.1.2.2 Probability distributions
    - 2.1.2.3 Densities
    - 2.1.2.4 Mean, variance and standard deviation
    - 2.1.2.5 The normal density function
    - 2.1.2.6 The binomial probability distribution

2.2 Limit Laws of Probability
- 2.2.1 The Law of Large Numbers (LLN)
  - 2.2.1.1 Independent random variables
  - 2.2.1.2 Identically distributed random variables
  - 2.2.1.3 Law of Large Numbers
- 2.2.2 The Central Limit Theorem (CLT)
  - 2.2.2.1 Examples (with MATLAB)
  - 2.2.2.2 Standardized sums
  - 2.2.2.3 Central Limit Theorem
- 2.2.3 Example: limit cycles in populations of neurons

2.3 Hypothesis Testing
- 2.3.1 Basic elements of a hypothesis test
  - 2.3.1.1 Main idea
  - 2.3.1.2 Definitions
  - 2.3.1.3 Critical region and significance level
- 2.3.2 Examples
  - 2.3.2.1 Voter preference
  - 2.3.2.2 Deployment of defibrillators
  - 2.3.2.3 Approximate tests using CLT
  - 2.3.2.4 Extreme fishing
  - 2.3.2.5 Poisson spiking

3. **Information Theory**

3.1 Measures of Information / Uncertainty
- 3.1.1 "Twenty questions"
  - 3.1.1.2 Bits
  - 3.1.1.3 Fractional bits
- 3.1.2 Entropy
  - 3.1.2.1 Example: uniform distribution
  - 3.1.2.2 Definition: discrete distribution
- 3.1.3 Lossless coding
  - 3.1.3.1 Codes
  - 3.1.3.2 Prefix codes
  - 3.1.3.3 Optimal prefix codes
- 3.1.4 Review
  - 3.1.4.1 Codes and trees
  - 3.1.4.2 Entropy
  - 3.1.4.3 Entropy and coding

3.2 Measures of Mutual Information
- 3.2.1 Pairs of random variables
  - 3.2.1.1 Joint probability distributions
  - 3.2.1.2 Joint entropies
  - 3.2.1.3 Conditional probability distributions
  - 3.2.1.4 Conditional entropies
- 3.2.2 Mutual information
- 3.2.3 Review
- 3.2.4 Example: neuroscience