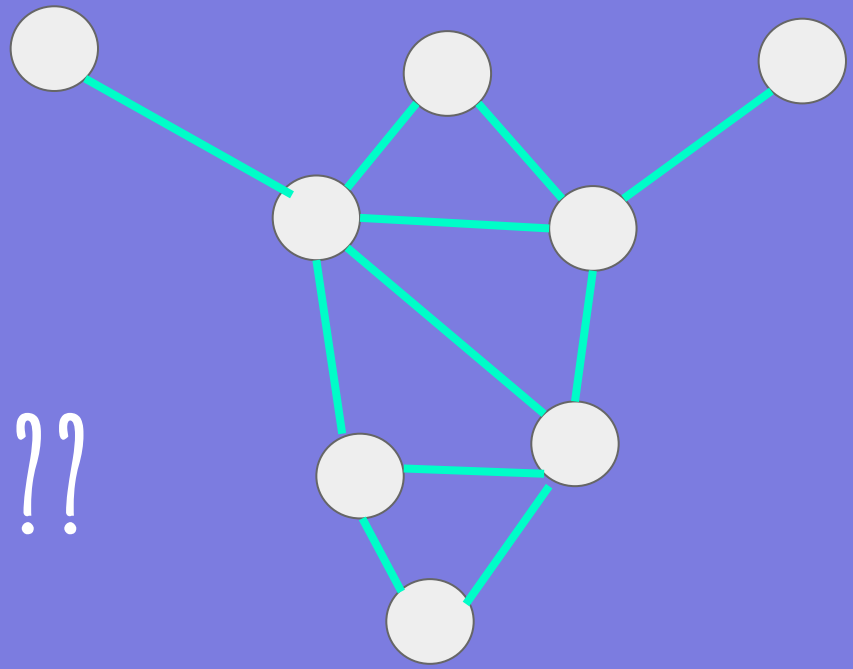
Fun with Graph Theory

Presented by Melissa McGuirl

Goals for Today

- Learn about graph theory
- Learn about the graph number and graph formula
- Learn about the color number
- Have fun with math

What is Graph theory??



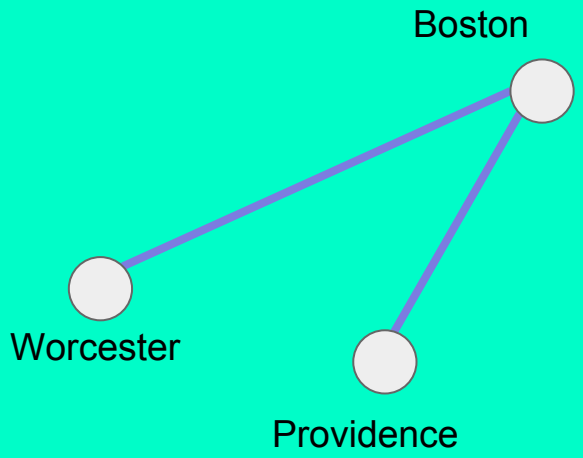
A Graph is a mathematical object consisting of Cities (Vertices) joined by Roads (straight edges).

Our Rules:

1. Each road must connect exactly two cities. We allow the graph of a single city with no roads.
2. Roads can not cross each other.
3. The graph is connected, meaning you can get from one city to any other city by traveling on the roads.

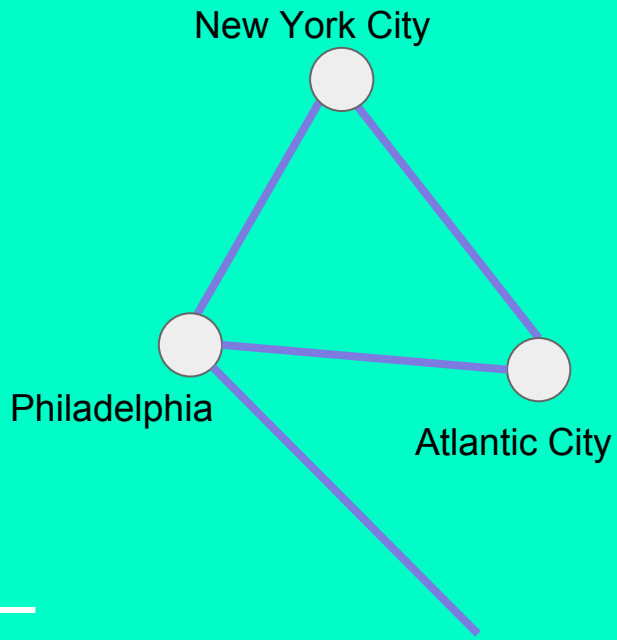
Is this a graph?

Why? Or why not?



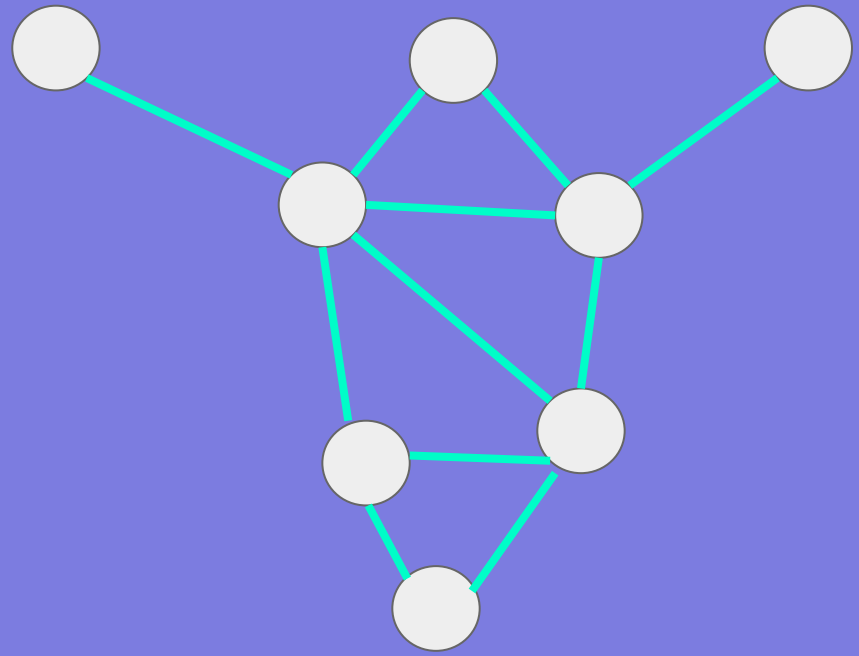
Is this a graph?

Why? Or why not?



This Graph has:

- 8 cities
- 11 roads
- 5 regions



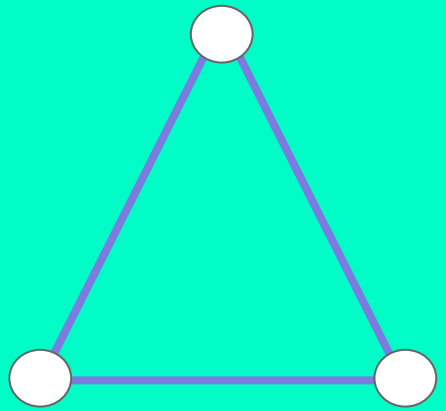
How can we characterize Graphs??

- Leonhard Euler, Swiss Mathematician (1707-1783)
- Studied Graph Theory
- Discovered something very interesting about connected, planar graphs when studying #Cities - #Roads + #Regions

(Number of cities) - (number of roads) + (Number of regions)

is called the Graph number of a graph.

Example 1

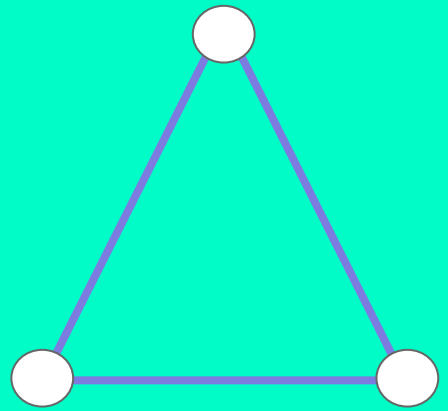


#Cities = ___
#Roads = ___
#Regions = ___

___

#Cities - #Roads + #Regions = ___

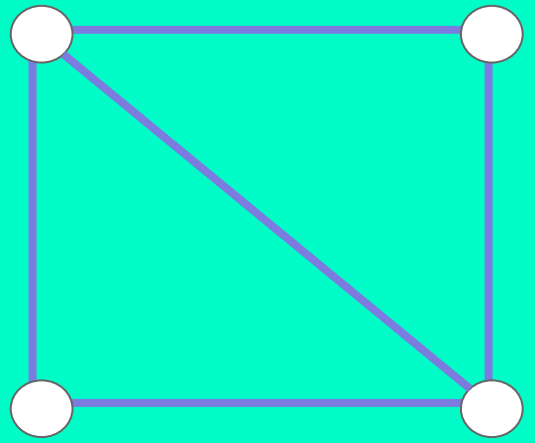
Example 1



#Cities = 3
#Roads = 3
#Regions = 2

#Cities - #Roads + #R = 2

Example 2

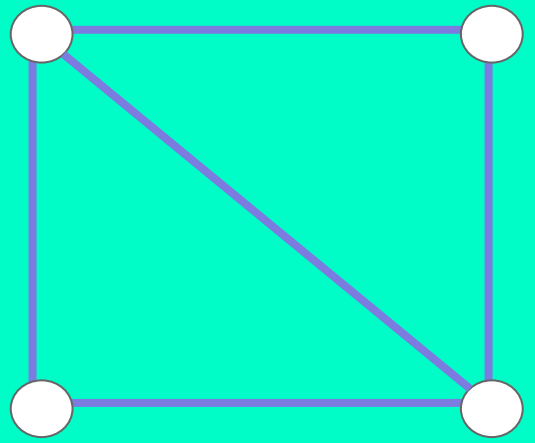


#Cities = ___
#Roads = ___
#Regions = ___

___

#Cities - #Roads + #Regions = ___

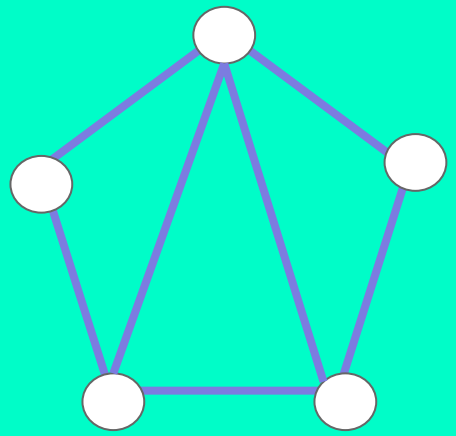
Example 2



#Cities = 4
#Roads = 5
#Regions = 3

#Cities - #Roads + #Regions = 2

Example 3

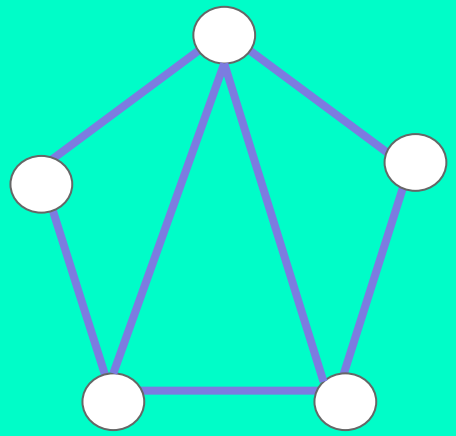


#Cities = ___
#Roads = ___
#Regions = ___

___

#Cities - #Roads + #Regions = ___

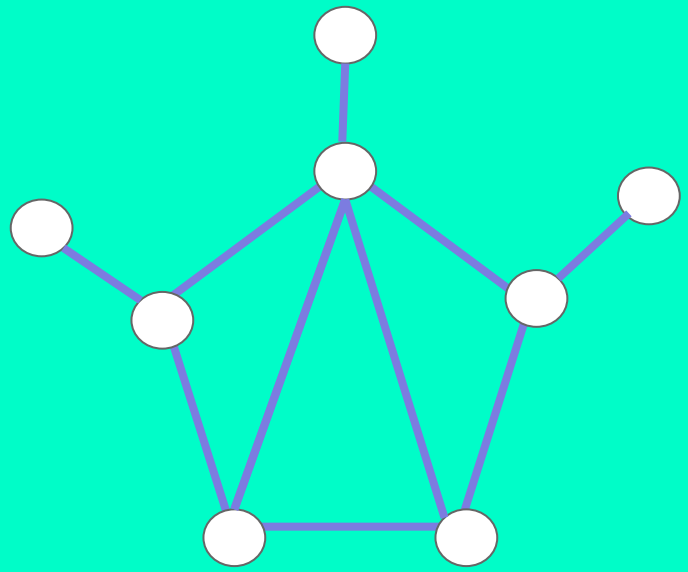
Example 3



#Cities = 5
#Roads = 7
#Regions = 4

#Cities - #Roads + #Regions = 2

Example 4

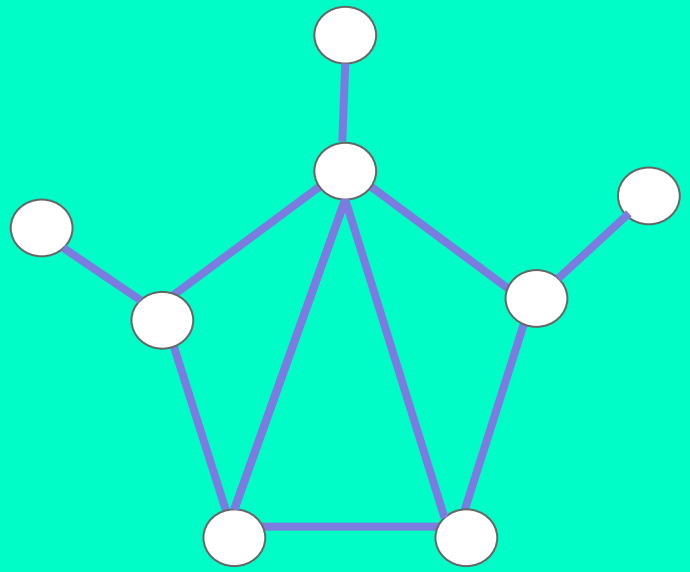


#Cities = ___
#Roads = ___
#Regions = ___

___

#Cities - #Roads + #Regions = ___

Example 4



#Cities = 8
#Roads = 10
#Regions = 4

#Cities - #Roads + #Regions = 2

Can you draw your own graph and compute its graph number?

What do you observe??

Graph Formula for connected flat graphs:

$$\#\text{cities} - \#\text{roads} + \#\text{Regions} = 2$$

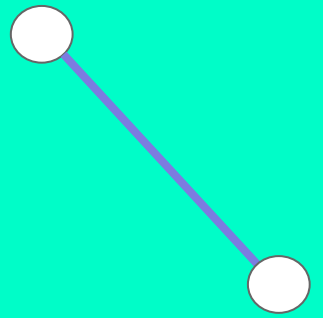
Is the graph Formula always true? Why?

Simplest graph

#Cities = 1
#Roads = 0
#Regions = 1

#Cities - #Roads + #Regions = 2

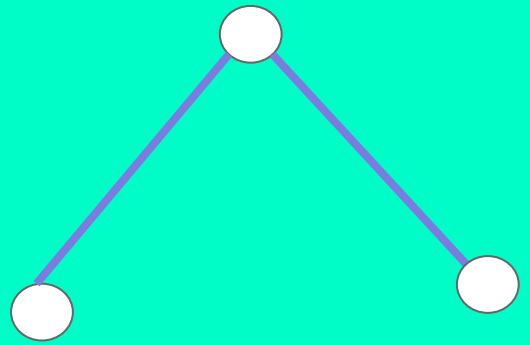
What if we add a connected city?



#Cities = 1 + 1 = 2
#Roads = 0 + 1 = 1
#Regions = 1

#Cities - #Roads + #Regions = 2

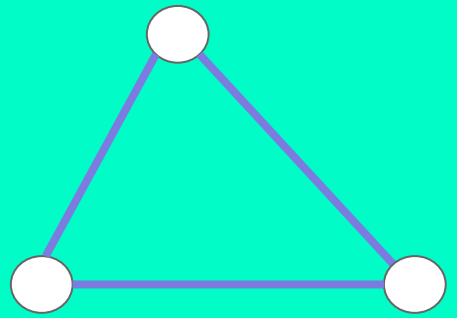
What if we add another connected City?



#Cities = 2 +1 = 3
#Roads = 1 + 1 = 2
#Regions = 1

#Cities - #Roads + #Regions = 2

What if we add a region?



#Cities = 3
#Roads = 3
#Regions = 2

#Cities - #Roads + #Regions = 2

- The graph formula holds for simplest graph of just 1 city
- Adding a connecting city or region does not change the graph number
- We conclude the graph formula holds for all connected flat graphs

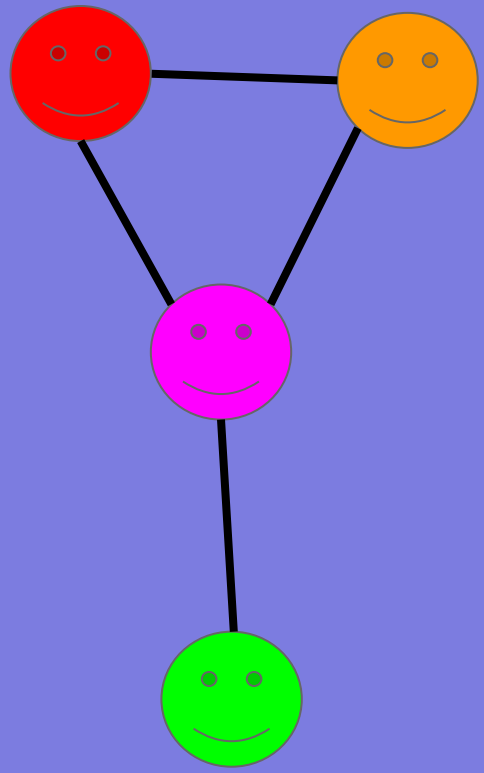
Now that we know what a graph is, let's experiment with colors!

Our Rules:

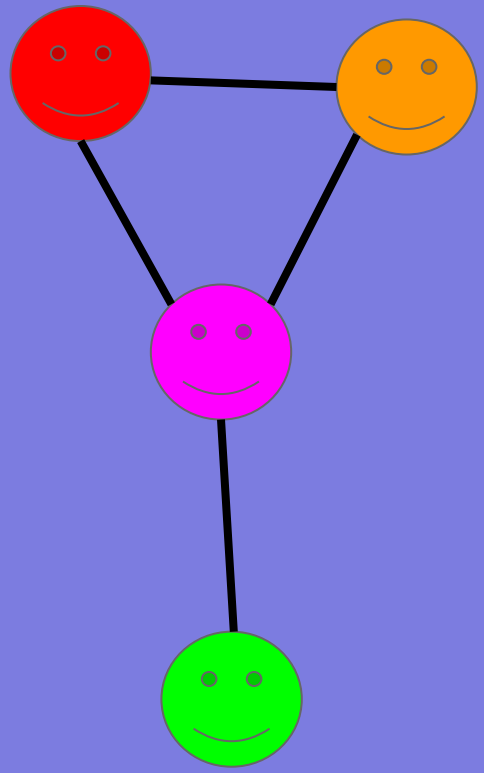
1. Each road must connect exactly two cities.
2. The graph is connected, meaning you can get from one city to any other city by traveling on the roads.

Color all the cities of your graph.

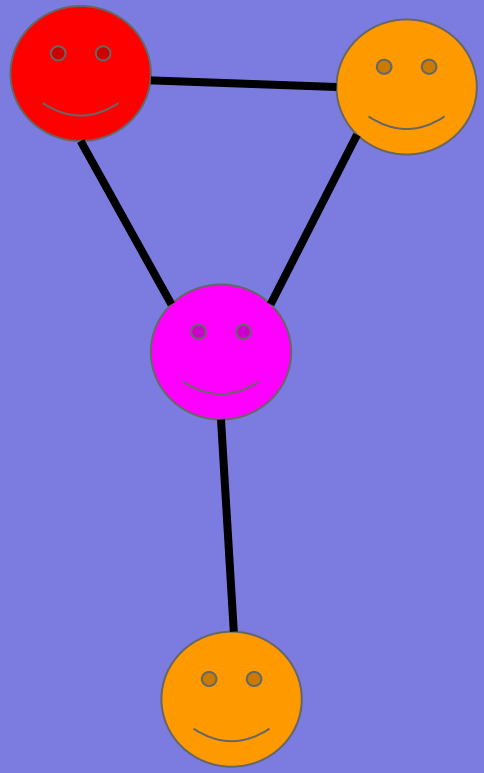
Rule: cities that share a road cannot have the same Color.



How many COLORS did we use? Can we do it with less?

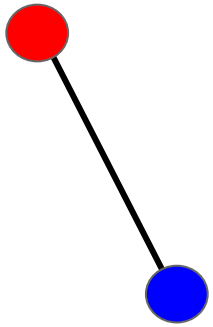


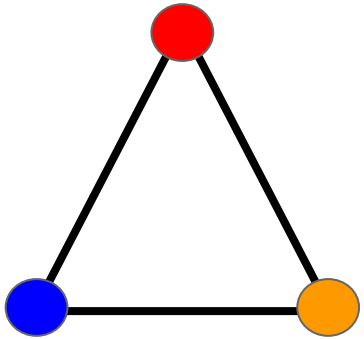
Here we only use 3 colors!

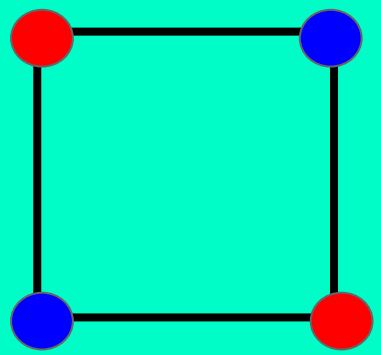


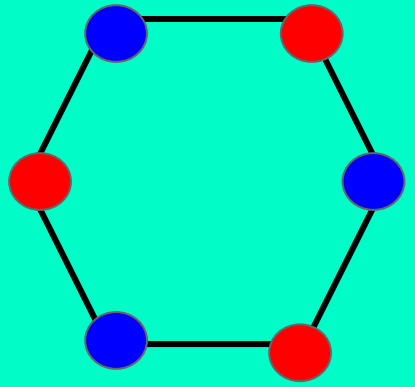
Challenge: Find the fewest number of colors Needed so that connected vertices have different colors.

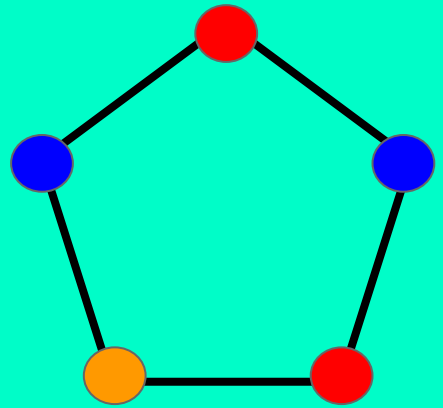
Example 6 : Cycle graphs

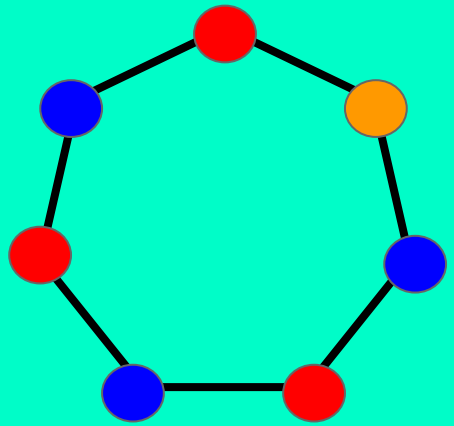






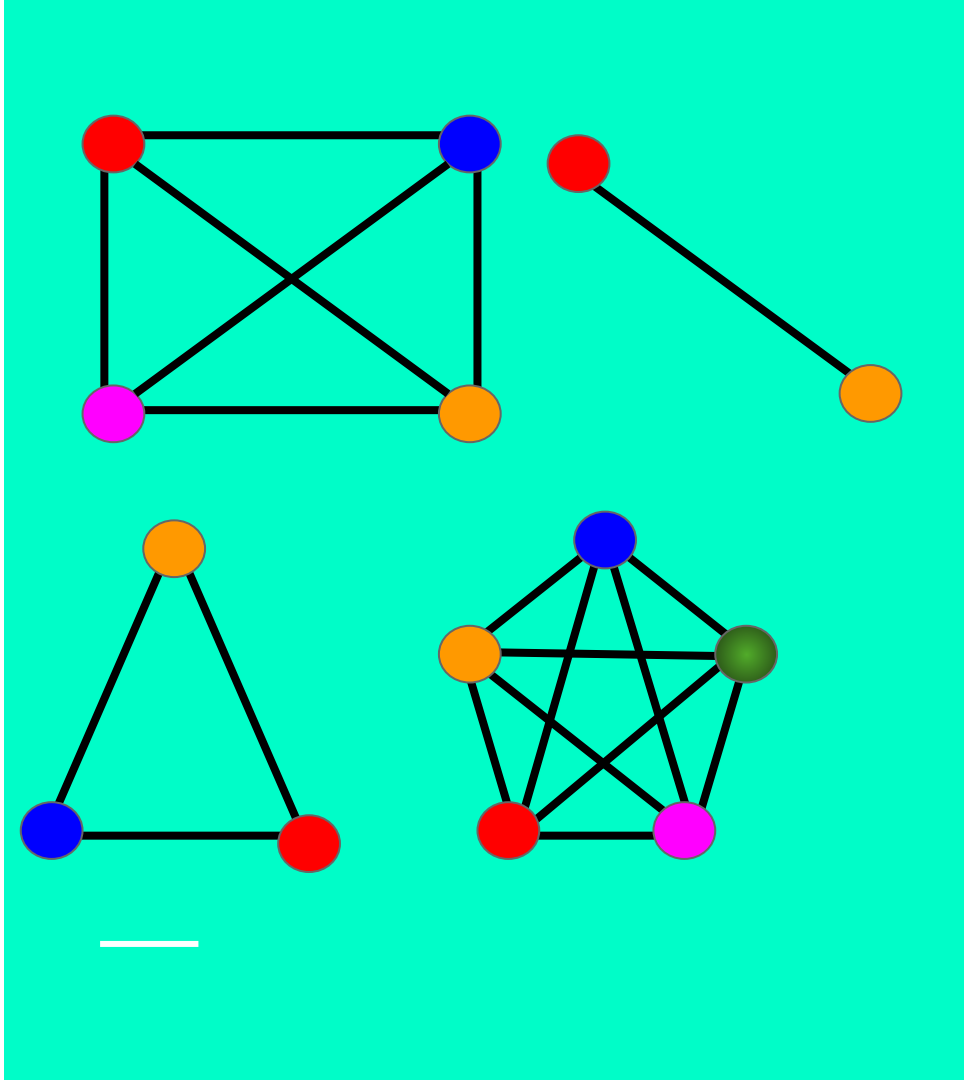






Example 7:

A complete graph is a graph where you can get from any city to any other city by a direct road



The color number of a graph is the smallest number of colors needed to color a graph by our rule.

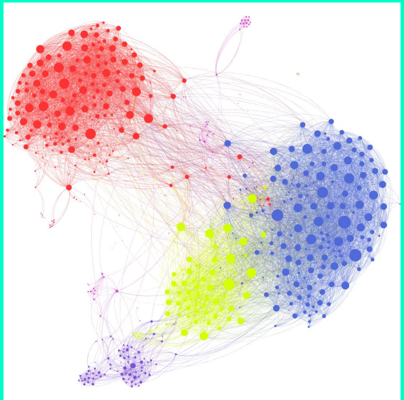
Find the color number of the graphs you generated earlier today.

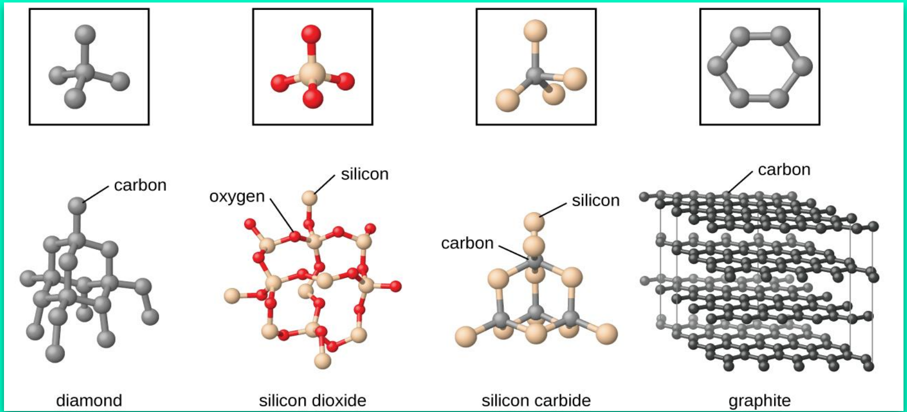
What do we know about the color number of a graph?

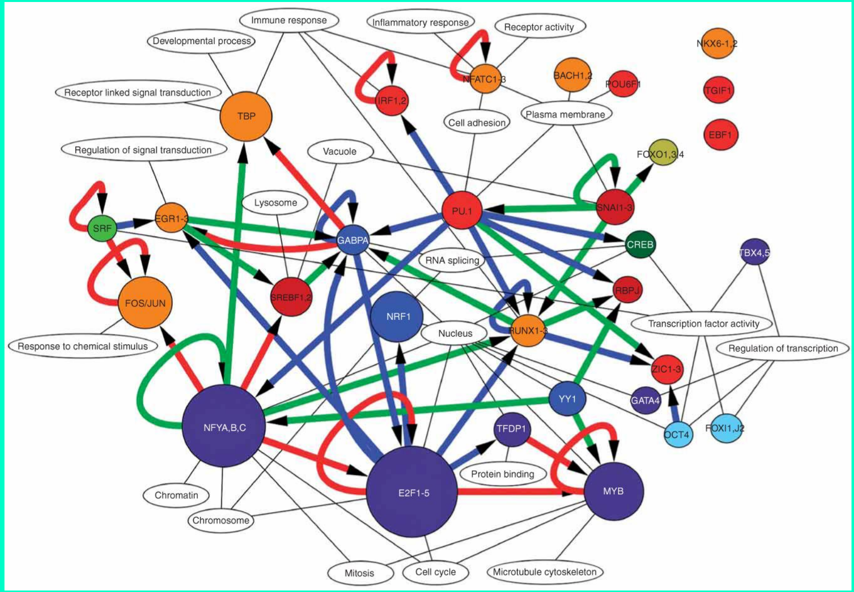
- The color number must be more than 1, unless the graph has no edges.
- The color number is less than or equal to the total number of cities in your graph, and for complete graphs the color number equals the number of cities in your graph.
- Cycle graphs have color number = 2 if the number of cities is even, and color number = 3 if the number of cities is odd.

What did we learn today?

- Definition of a graph
- Applications of graph theory
- The Euler characteristic (graph number)
- Euler’s formula (graph formula)
- Chromatic number (color number)




carbon
oxygen
silicon
silicon
carbon
carbon
diamond
silicon dioxide
silicon carbide
graphite


Immune response
Inflammatory response
Receptor activity
Developmental process
Receptor linked signal transduction
TBP
IRF1,2
NFATC1-3
BACH1,2
POU6F1
NKX6-1,2
TGIF1
EBF1
Cell adhesion
Plasma membrane
Regulation of signal transduction
Vacuole
FOXO1,3,4
Lysosome
PU.1
SNAI1-3
SRF
EGR1-3
GABPA
CREB
RNA splicing
TBX4,5
SREBF1,2
RBPJ
FOS/JUN
NRF1
Nucleus
RUNX1-3
Transcription factor activity
Response to chemical stimulus
Regulation of transcription
ZIC1-3
YY1
GATA4
NFYA,B,C
TFDP1
OCT4
FOXI1,J2
Protein binding
E2F1-5
MYB
Chromatin
Chromosome
Mitosis
Cell cycle
Microtubule cytoskeleton

Thanks for listening!!
Questions??

Most of the material in this presentation comes from Joel David Hamkins:

- http://jdh.hamkins.org/math-for-eight-year-olds/
- http://jdh.hamkins.org/math-for-seven-year-olds-graph-coloring-chromatic-numbers-eulerian-paths/