HW1, due Sept. 15

1. Suppose $f : [0,1] \to [0,1]$ is C^2 and has a fixed point x_*. Suppose $|f'(x_*)| < 1$. Prove that x_* is attracting– that is, there exists an interval J containing x_* such that $x_0 \in J$ implies $x_n \to x_*$.

The following questions apply to the logistic map.

2. Prove that the origin is attracting for $0 < r < 1$, and undergoes a transcritical bifurcation at $r = 1$.

3. Show that the fixed point $x = 1 - 1/r$ undergoes a first period-doubling bifurcation at $r = 3$, and a second period-doubling bifurcation at $r = 1+\sqrt{6}$.

4. Compute the first two superstable orbits and parameter values R_0 and R_1.

5. There are several smooth dark curves running through the orbit diagram of the logistic map (Fig. 1.1). What are these curves?

6. Consider the Feigenbaum-Cvitanović functional equation. Compute a 2-nd order approximate solution by assuming $g(x) \approx 1 + c_2 x^2$. If you are more ambitious, try a fourth-order approximation.