Linear Algebra

MA 242 (Spring 2013)
Instructor: M. Chirilus-Bruckner

LU DECOMPOSITION

– Solving linear systems by matrix factorization –

Assume you are given a factorization of am $m-by-n$ matrix A as $A=LU$ where L is an $m-by-m$ lower triangular matrix and U is an $m-by-n$ upper triangular matrix. For instance,

$$A=\underbrace{\begin{bmatrix}1&0&0&0\\ *&1&0&0\\ *&*&1&0\\ *&*&*&1\end{bmatrix}}_{=L}\underbrace{\begin{bmatrix}\square&*&*&*&*\\ 0&\square&*&*&*\\ 0&0&0&\square&*\\ 0&0&0&0&0\end{bmatrix}}_{=U}.$$

Then

$$Ax=L\underbrace{(Ux)}_{=y}=b$$

can be broken down into two "simpler" systems

$$\begin{aligned}.\qquad Ly&=b,\\ Ux&=y.\end{aligned}$$

This is of advantage, for instance, if one has to solve $Ax=b$ with many different RHSs b, since solving these two systems by substitution is much less expensive than the full Gauss elimination.

- *Historical note*:
  LU-decomposition was introduced by Alan Turing (see back of this page).
- *Software*:
  LU-decomposition is part of the numerical linear algebra software library LAPACK (which is used for instance by MATLAB or Mathematica)
- *Easy example illustrating how to construct L and U from A:*

Alan Turing

– father of computer science and artificial intelligence –

Alan Turing was born on 23 June, 1912, in London. His father was in the Indian Civil Service and Turing's parents lived in India until his father's retirement in 1926. Turing and his brother stayed with friends and relatives in England. Turing studied mathematics at Cambridge University, and subsequently taught there, working in the burgeoning world of quantum mechanics. It was at Cambridge that he developed the proof which states that automatic computation cannot solve all mathematical problems. This concept, also known as the Turing machine, is considered the basis for the modern theory of computation.

In 1936, Turing went to Princeton University in America, returning to England in 1938. He began to work secretly part-time for the British cryptanalytic department, the Government Code and Cypher School. On the outbreak of war he took up full-time work at its headquarters, Bletchley Park.

Here he played a vital role in deciphering the messages encrypted by the German Enigma machine, which provided vital intelligence for the Allies. He took the lead in a team that designed a machine known as a bombe that successfully decoded German messages. He became a well-known and rather eccentric figure at Bletchley.

After the war, Turing turned his thoughts to the development of a machine that would logically process information. He worked first for the National Physical Laboratory (1945-1948). His plans were dismissed by his colleagues and the lab lost out on being the first to design a digital computer. It is thought that Turing's blueprint would have secured them the honour, as his machine was capable of computation speeds higher than the others. In 1949, he went to Manchester University where he directed the computing laboratory and developed a body of work that helped to form the basis for the field of artificial intelligence. In 1951 he was elected a fellow of the Royal Society.

. http://www.bbc.co.uk/history/people/alan_turing