Linear Algebra

MA 242   (Spring 2013)
Instructor: M. Chirilus-Bruckner

Invertible Matrix Theorem
– confluence of different concepts –

Let A be a **square matrix** of size $n \times n$. Then the following statements are equivalent.

1. A is invertible.

2. A is row equivalent to the identity matrix.

3. A has n pivot positions.

4. The equation A has only the trivial solution.

5. The columns of A form a linearly independent set.

6. The linear transformation $x \mapsto Ax$ is one-to-one.

7. The equation $Ax = b$ has at least one solution for each $b \in \mathbb{R}^n$.

8. The columns of A span $\mathbb{R}^n$.

9. The linear transformation maps $\mathbb{R}^n$ onto $\mathbb{R}^n$.

10. There is an $n \times n$ matrix C such that $CA = I$.

11. There is an $n \times n$ matrix D such that $AD = I$.

12. A^T is invertible.