Linear Algebra

MA 242 (Spring 2013)
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Gram-Schmidt process

— Computing an orthogonal basis —

Given linearly independent vectors $u_1, \dots, u_p$ compute an orthogonal set $v_1, \dots, v_p$ that is an orthogonal basis for $H = \text{Span}\{u_1, \dots, u_p\}$:

1. $v_1 = u_1$
2. $v_2 = u_2 - \left(\frac{u_2 \cdot v_1}{v_1 \cdot v_1}\right) v_1$
3. $v_3 = u_3 - \left(\frac{u_3 \cdot v_1}{v_1 \cdot v_1}\right) v_1 - \left(\frac{u_2 \cdot v_2}{v_2 \cdot v_2}\right) v_2$
4. $v_4 = u_4 - \left(\frac{u_4 \cdot v_1}{v_1 \cdot v_1}\right) v_1 - \left(\frac{u_4 \cdot v_2}{v_2 \cdot v_2}\right) v_2 - \left(\frac{u_4 \cdot v_3}{v_3 \cdot v_3}\right) v_3$

   $\vdots$
5. $v_p = u_p - \left(\frac{u_p \cdot v_1}{v_1 \cdot v_1}\right) v_1 - \dots\dots\dots\dots\dots\dots - \left(\frac{u_p \cdot v_{p-1}}{v_{p-1} \cdot v_{p-1}}\right) v_{p-1}$