Linear Algebra

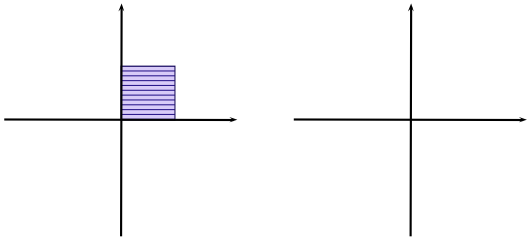
MA 242 (Spring 2013)
Instructor: M. Chirilus-Bruckner

Linear transformations

– reflections, rotations, contractions, expansions, shears and projections –

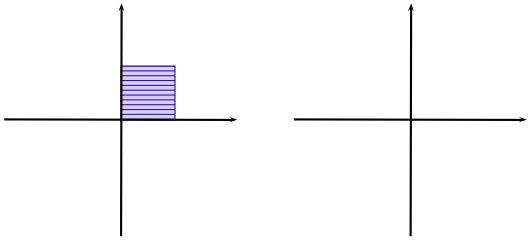
Illustrate the action of the linear transformation $T(x) = Ax$.

- $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$



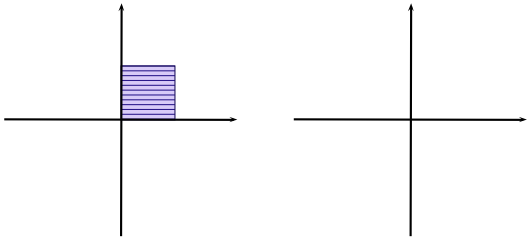
description:

- $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$



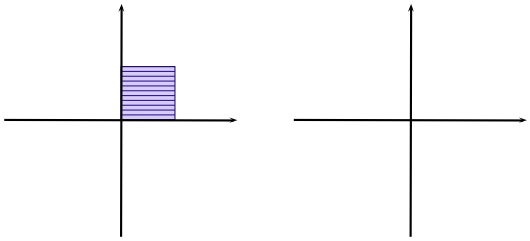
description:

- $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$



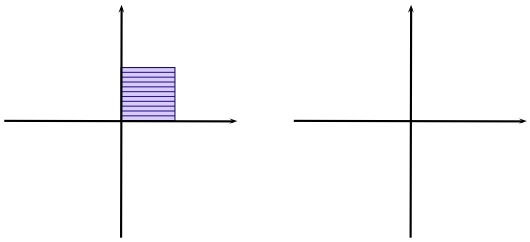
description:

- $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$



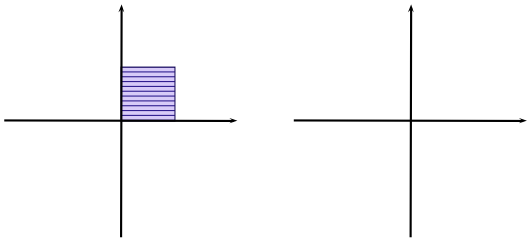
description:

- $A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$



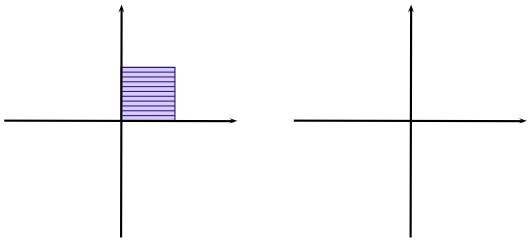
description:

- $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$



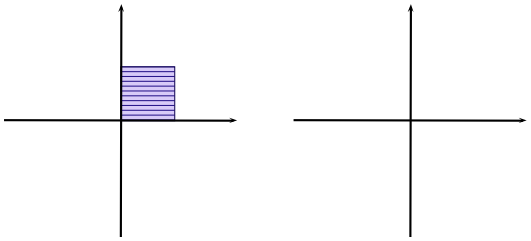
description:

- $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$



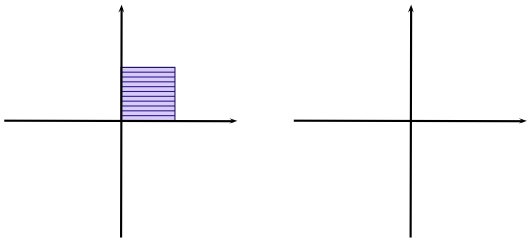
description:

- $A = \begin{bmatrix} \cos(\varphi) & -\sin(\varphi) \\ \sin(\varphi) & \cos(\varphi) \end{bmatrix}$



description:

- $A = \begin{bmatrix} h & 0 \\ 0 & 1 \end{bmatrix}$

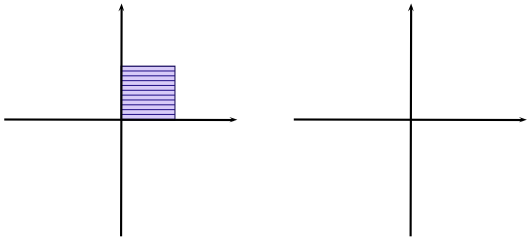


description:

What is the difference between $0 < h < 1$ and $h > 1$?

What is the matrix for the vertical version of this transformation?

- $A = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$

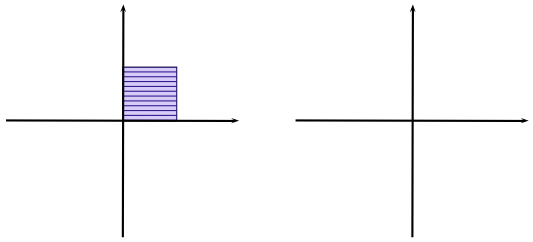


description:

What is the difference between $k > 0$ and $k < 0$?

What is the matrix for the vertical version of this transformation?

- $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$



description:

What is the matrix for the horizontal version of this transformation?

An example in $\mathbb{R}^3$

1. Illustrate the transformation that is obtained by executing A_1 then A_2 and lastly A_3 by using the **object on the next page**.

$$A_1 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2. Describe its action in words.

3. Compute the product

$$A_3 A_2 A_1 =$$

4. Motivate why matrix multiplication is not commutative by illustrating the action of the transformation $A_1 A_3$ versus the transformation $A_3 A_1$.