

A **fractal** (*fractus* Latin for *broken, uneven*) is an object or quantity that displays self-similarity, i.e., for which any suitably chosen part is similar in shape to a given larger or smaller part when magnified or reduced to the same size. The object need not exhibit exactly the same structure at all scales, but the same "type" of structures must appear on all scales.

sources: <http://mathworld.wolfram.com/Fractal.html> and <http://www.merriam-webster.com/dictionary/fractal>

The **Barnsley fern** is an example of a fractal that is created by an iterated function system, in which a point (the seed or pre-image) is repeatedly transformed by using one of four transformation functions. A random process determines which transformation function is used at each step. The final image emerges as the iterations continue. The transformations are affine transformations T_1, \dots, T_4 of the form

$$T_j(x, y) = A_j \begin{bmatrix} x \\ y \end{bmatrix} + v_j$$

where A_j is a 2×2 -matrix and v_j is a vector.

An **affine transformation** is any transformation that preserves collinearity (i.e., all points lying on a line initially still lie on a line after transformation) and ratios of distances (e.g., the midpoint of a line segment remains the midpoint after transformation). In general, an affine transformation is a composition of rotations, translations, dilations, and shears. While an affine transformation preserves proportions on lines, it does not necessarily preserve angles or lengths. Any triangle can be transformed into any other by an affine transformation, so all triangles are affine and, in this sense, affine is a generalization of congruent and similar.

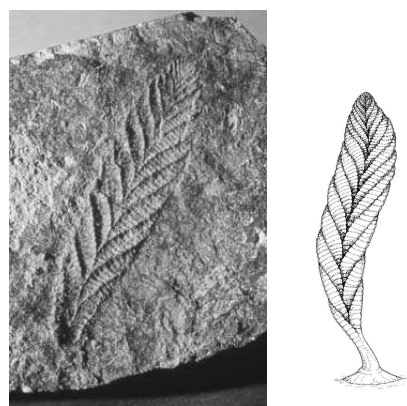
source: <http://mathworld.wolfram.com/AffineTransformation.html>



Figure created with MATLAB code (see next page)

Charnia. Fractal structures are ubiquitous in nature. They are believed to have provided a construction mechanism for the first forms of life in the early history of evolution. An animal that is believed to have lived in the Ediacaran period of the Pre-Cambrian era (roughly 600000 million years ago) is *Charnia* (named after Charnwood Forest in Leicestershire, England, where the first fossilised specimen was found). The species is a type of fractal life form that grew on the sea floor and is believed to have fed on nutrients in the water. Despite *Charnia*'s fern-like appearance, it is not a plant as it lived deep down in the ocean floor, which had little sunlight for photosynthesis to occur.

source: <http://en.wikipedia.org/wiki/Charnia>



fossil and a life restoration (Mark Evans)

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% Matlab code for a beautiful Fractal Fern
% the IFS defined in Table 3.8.3 of Michael Barnsley's "Fractals Everywhere"
% AUTHOR: Elizabeth Doman Copene
%
% just type "fern" in the Matlab command window to run it
clear all
iterations=100000; %the number of iterations
% recall, the transformations are of the form A*[x,y]+t
% and, there are four such transformations ...
A1=[0 0 ; 0 0.16];
A2=[0.85 0.04 ; -0.04 0.85];
A3=[0.2 -0.26 ; 0.23 0.22];
A4=[-0.15 0.28 ; 0.26 0.24];
t1=[0 ; 0];
t2=[0 ; 1.6];
t3=[0 ; 1.6];
t4=[0 ; 0.44];
% here are the probabilities with which each transformation is applied
% notice that p1+p2+p3+p4=1
p1=0.01;
p2=0.85;
p3=0.07;
p4=0.07;

% the initial point
x(1)=0;
y(1)=0;
% but, let's write the (x,y) points as a vector, v
v=[x(1);y(1)]; % here is the initial point
for n=2:iterations
    % choose a random number, k, between 0 and 1
    k=rand;
    % depending on your random number ...
    % do one of the four transformations to get a new point
    if k<p1
        v=A1*v+t1;
    elseif k<p1+p2
        v=A2*v+t2;
    elseif k<p1+p2+p3
        v=A3*v+t3;
    else
        v=A4*v+t4;
    end
    % now, go back and define your (x,y) point as elements of the vector v
    x(n)=v(1);
    y(n)=v(2);
end
%now, let's plot all those (x,y) points that we just computed!
opengl software %this is a fix to ensure that Matlab won't crash :)
hold off
plot(x,y, '.', 'Color', [0.1 0.3 0], 'Markersize', 1)
% initial point in red
hold on
plot(x(1),y(1), 'r*', 'Markersize', 10)
axis('equal', 'off');
set(gcf, 'color', 'white');

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