

LINEAR ALGEBRA

— MA 242 —

Exercise Sheet

4

– not graded –

- 1** Given a linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$, $x \mapsto Ax$ with $A \in \mathbb{R}^{2 \times 2}$, complete the following formula where D is a region in \mathbb{R}^2 (with finite area) and $T(D)$ its image under T :

$$\text{“area of } T(D)\text{”} = \underline{\hspace{2cm}} \text{ “area of } D \text{”}$$

- 2** Let S be a parallelogram determined by the points

$$(1, 1), (3, 1), (2, 2), (4, 2)$$

and consider a linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by

$$T(x_1, x_2) = [2x_1 - 3x_2, x_1 + 4x_2].$$

- a) Compute the area of S .
- b) Compute the area of $T(S)$ *without* using the formula from **1**.
- c) Compute the area of $T(S)$ *via* the formula from **1**.

- 3** Which of the following are subspaces of \mathbb{R}^3 ? Explain.

- a) $\{[2\lambda, 3\lambda, \lambda] \mid \lambda \in \mathbb{R}\}$
- b) $\{[2\lambda, 3\lambda, 1] \mid \lambda \in \mathbb{R}\}$
- c) $\{[2x_1 - x_2, 3x_2 + 1, x_1] \mid x_1, x_2 \in \mathbb{R}\}$
- d) $\{[y_1, y_2, y_3] \mid -17y_3 + 2y_1 + 1 - y_2 = 0, 2y_3 + y_1 + y_2 = 0\}$

- 4** Given $A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 1 \\ 7 & 1 & 1 \end{bmatrix}$ and a parameter λ .

- a) Compute $\det(A - \lambda I_3)$.
- b) For which λ is the matrix $A - \lambda I_3$ *not* invertible?
- c) For which λ does the equation $Ax = \lambda x$ have *non-trivial* solutions?

2 a, 2. 2 b, c 22 (note: $T(x) = Ax$ with $\det(A) = 11$), 3 a yes, 3 b, c, d no, 4 b, c $\lambda \in \{0, 1, 2\}$