Linear Algebra

— MA 242 —

Extra Credit

HW 9

1 (Characteristic polynomial, determinants and traces)

Consider the square matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

a) Compute the characteristic polynomial for A.

b) The trace, $\mathrm{tr}(A)$, of a square matrix A is defined as the sum of its diagonal entries. For the 2×2 matrix A we have $\mathrm{tr}(A) = a + d$. How are the coefficients of the characteristic polynomial computed in **a)** related to $\mathrm{tr}(A)$ and $\det(A)$?

c) Show that for the 2×2 matrix A we have that both eigenvalues are real if and only if $\det(A) \le \left(\frac{1}{2}\,\mathrm{tr}(A)\right)^2$.

2 (Block matrices)

Consider the block matrix $\mathcal{D} = \begin{bmatrix} A & X \\ 0 & B \end{bmatrix}$ where A, B, X are matrices and 0 is a zero matrix.

a) It holds that

$$\det(\mathcal{D}) = \det(A)\det(B).$$

Explain why this is plausible.

b) Assume that the characteristic polynomial of A is $p_A(\lambda)$ and the characteristic polynomial of B is $p_B(\lambda)$. Given the formula in **a)**, conclude that the characteristic polynomial for $\mathcal{D}$ is $p_{\mathcal{D}}(\lambda) = p_A(\lambda)p_B(\lambda)$.

c) Determine eigenvalues and determinant of the following matrix.

$$A = \begin{bmatrix} 1 & 0 & 0 & 7 & -11 & \pi \\ 37 & 2 & 0 & 8 & 44 & 22 \\ 5 & \pi & 3 & 9 & 102 & 19 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 3 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 2 & 1 & \frac{1}{3} \end{bmatrix}$$