LINEAR ALGEBRA
— MA 242 —

Extra Credit

HW 12

(Householder reflection)

Consider a vector

$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \in \mathbb{R}^3$$

with $u^T u = 1$.

a) Compute the matrices $P = uu^T, Q = Id - 2P$.

b) Use the rules for transposition to show that $P^2 = P$, $P^T = P$ and $Q^2 = Id$.

c) The transformation $x \mapsto Qx$ is called **Householder reflection**. To illustrate this name use

$$u = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad x = \begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix},$$

to compute P and Q and then determine Px, Qx and $x - Px$. Make a sketch that illustrates how x is reflected through the $x_1 x_2$-plane.