Linear Algebra

— MA 242 —

Exercise Sheet

6

– not graded –

Consider

$$u = \begin{bmatrix} -1 \\ 3 \end{bmatrix}, \quad v = \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \quad b_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad b_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

1 Compute the following quantities (with the notation $u \cdot v := u^T v$ for the inner product).

$a)$ $b_1 \cdot b_1$ = ______________________ $e)$ $\frac{u \cdot b_2}{b_2 \cdot b_2} b_2$ = ______________________

$b)$ $b_2 \cdot b_2$ = ______________________ $f)$ $\|u\|$ = ______________________

$c)$ $b_1 \cdot b_2$ = ______________________ $g)$ $\|u - v\|$ = ______________________

$d)$ $\frac{u \cdot b_1}{b_1 \cdot b_1} b_1$ = ______________________ $h)$ $\|v - u\|$ = ______________________

2 Compute the coordinate representation $[u]_{\mathcal{B}}$ of u w.r.t. the basis $\mathcal{B} = \{b_1, b_2\}$.

$\mathcal{P}_{\mathcal{B}}^{-1} =$ ______________________, $[u]_{\mathcal{B}} =$ ______________________

Check your answer by using that

$$u = \frac{u \cdot b_1}{b_1 \cdot b_1} b_1 + \frac{u \cdot b_2}{b_2 \cdot b_2} b_2,$$

and your computations from **1**.

3 Give two vectors in $\mathbb{R}^3$ that are orthogonal (and are not columns of an identity matrix).