BU Department of Mathematics & Statistics **Instructor**: M. Chirilus-Bruckner

Linear Algebra

— MA 242 —

Exercise Sheet

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– not graded –

1 Consider a matrix $A \in \mathbb{R}^{3\times 4}$ given by $A = \begin{bmatrix} 1 & 1.7 & -1 & 22 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{bmatrix}$.

a) Determine the dimension of $\mathrm{Col}(A)$.

b) Determine the dimension of $\mathrm{Nul}(A)$.

c) How many pivot columns does A have?

d) How many non-pivot columns does A have? How many free variables does $Ax = 0$ have?

e) Is there a relation between **a)-d)** and the dimension of A ?

2 Consider the sets $\mathcal{B} = \left\{ \begin{bmatrix} 7 \\ 5 \end{bmatrix}, \begin{bmatrix} -3 \\ -1 \end{bmatrix} \right\}$ and $\mathcal{C} = \left\{ \begin{bmatrix} 1 \\ -5 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \end{bmatrix} \right\}$ and the vector $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

a) Why is $\mathcal{B}$ a basis for $\mathbb{R}^2$? Why is $\mathcal{C}$ a basis for $\mathbb{R}^2$? Give another valid choice of basis.

b) Compile the change of variables matrices $\mathcal{P}_\mathcal{B}$ and $\mathcal{P}_\mathcal{C}$ associated with $\mathcal{B}$ and $\mathcal{C}$. Why are these matrices invertible?

c) Compute the coordinate representation $[x]_\mathcal{B}$ of x w.r.t. $\mathcal{B}$ and $[x]_\mathcal{C}$ of x w.r.t. $\mathcal{C}$.

d) Change of basis between $\mathcal{B}$ and $\mathcal{C}$: Noting that $[x]_\mathcal{B} = \mathcal{P}_\mathcal{B}^{-1} x$ and $[x]_\mathcal{C} = \mathcal{P}_\mathcal{C}^{-1} x$, we have

$$[x]_\mathcal{C} = \mathcal{P}_\mathcal{C}^{-1} \mathcal{P}_\mathcal{B}\, [x]_\mathcal{B}.$$

Why? Verify this using your answers from **c)**.