Linear Algebra

— MA 242 —

Exercise Sheet

2

1.1 Execute the following matrix-vector multiplications, if they are defined.

$$\begin{bmatrix} 1 & -\sqrt{17} \\ \sqrt{2} & 3 \end{bmatrix} \begin{bmatrix} 7 \\ -9 \end{bmatrix} = \begin{bmatrix} 7+9\sqrt{17} \\ 7\sqrt{2}-27 \end{bmatrix}, \qquad \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} = \text{not defined},$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & -1 \end{bmatrix} = \text{not defined}, \qquad \begin{bmatrix} 7 & 8 & 9 & 10 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} = -2,$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \\ -2 \\ -2 \end{bmatrix}, \qquad \begin{bmatrix} 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \end{bmatrix} = \text{not defined},$$

1.2 Define linear independence and linear dependence relations.

If $S = \{v_1, \ldots, v_n\}$ is a set of vectors, then the vector equation

$$c_1 v_1 + \ldots c_n v_n = 0 \tag{1}$$

has at least one solution, namely

$$c_1 = 0, c_2 = 0, \ldots, c_n = 0.$$

If this is the only solution, then S is called a **linearly independent** set. If there are other solutions, then S is called a **linearly dependent** set and (1) is called **linear dependence relation**.

There are more questions on the next page!

1.3 $\left[\begin{array}{ccc|c} 3 & -6 & 9 & 0 \\ -6 & 4 & h & 0 \\ 1 & -3 & 3 & 0 \end{array}\right]$ For which $h \in \mathbb{R}$ does the corresp. system of linear equations have $a)$ one solution, $b)$ no solution, or $c)$ infinitely many solutions?

$$\left[\begin{array}{ccc|c} 3 & -6 & 9 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 18+h & 0 \end{array}\right]$$

This system always has the trivial solution $[0,0,0]$, so $b)$ never happens. For $h \neq -18$ this is the only solution, for $h = -18$ there are infinitely many solutions.

1.4 For which values of $h \in \mathbb{R}$ are the vectors below linearly independent?

$$\begin{bmatrix} 3 \\ -6 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} -6 \\ 4 \\ -3 \end{bmatrix}, \quad \begin{bmatrix} 9 \\ h \\ 3 \end{bmatrix}$$

Checking for linear independence gives the vector equation

$$c_1 \begin{bmatrix} 3 \\ -6 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} -6 \\ 4 \\ -3 \end{bmatrix} + c_3 \begin{bmatrix} 9 \\ h \\ 3 \end{bmatrix} = 0,$$

whose augmented matrix coincides with the one in **1.3**. Hence, for $h \neq -18$ the vectors are linearly independent.

1.5 Write the solution set for the linear equation $2x_1 + 6x_2 + 8x_3 = 0$ in parametric form. What geometric object can be associated with the solution set?

$$2x_1 + 6x_2 + 8x_3 = 0 \Leftrightarrow x_1 + 3x_2 + 4x_3 = 0 \Leftrightarrow x_1 = -3x_2 - 4x_3$$

Solutions are of the form

$$x = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -3b - 4c \\ b \\ c \end{bmatrix} = b \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix} + c \begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix}, \qquad b, c \in \mathbb{R},$$

so they all lie in a plane through the origin.

– The End –