Millersville University
Department of Mathematics
MATH 161, *Calculus I*, Test 2
October 13, 2006, 08:00-08:50AM

Name Answer Key

Please answer the following questions. Your answers will be evaluated on their correctness, completeness, and use of mathematical concepts we have covered. Please show all work and write out your work neatly. Answers without supporting work will receive no credit. The point values of the problems are listed in parentheses.

1. (10 points each) Find the derivatives of the following functions. You do not need to simplify your results.

   (a) $f(x) = (x+2)^7(x+3)^8$

$$f'(x) = \left[\frac{d}{dx}(x+2)^7\right](x+3)^8 + (x+2)^7\left[\frac{d}{dx}(x+3)^8\right]$$

$$= 7(x+2)^6(x+3)^8 + (x+2)^7 8(x+3)^7$$

   (b) $f(x) = \frac{x}{\sqrt{9-4x}}$

$$f'(x) = \frac{\left[\frac{d}{dx}(x)\right]\sqrt{9-4x} - x\left[\frac{d}{dx}(9-4x)^{1/2}\right]}{\left(\sqrt{9-4x}\right)^2}$$

$$= \frac{\sqrt{9-4x} - x \cdot \frac{1}{2}(9-4x)^{-1/2}(-4)}{9-4x}$$

(c) $f(x) = \dfrac{\sin x}{\sqrt[3]{1+\sqrt{x}}}$

$$f'(x) = \frac{\left[\frac{d}{dx}\sin x\right]\sqrt[3]{1+\sqrt{x}} - \sin x\left[\frac{d}{dx}(1+x^{1/2})^{1/3}\right]}{\left(\sqrt[3]{1+\sqrt{x}}\right)^2}$$

$$= \frac{(\cos x)\sqrt[3]{1+\sqrt{x}} - (\sin x)\frac{1}{3}(1+x^{1/2})^{-2/3}\frac{1}{2}x^{-1/2}}{(1+\sqrt{x})^{2/3}}$$

(d) $f(x) = \tan^{-1}\left(\sqrt{1+x^2}\right)$

$$f'(x) = \frac{1}{1+\left(\sqrt{1+x^2}\right)^2} \cdot \frac{d}{dx}\left[(1+x^2)^{1/2}\right]$$

$$= \frac{1}{1+1+x^2} \; \frac{1}{2}(1+x^2)^{-1/2}\left[\frac{d}{dx}(1+x^2)\right]$$

$$= \frac{1}{2+x^2} \; \frac{1}{2}(1+x^2)^{-1/2} \, 2x$$

(e) $f(x) = \dfrac{e^{2x}}{1+e^x}$

$$f'(x) = \frac{\left[\frac{d}{dx}e^{2x}\right](1+e^x) - e^{2x}\left[\frac{d}{dx}(1+e^x)\right]}{(1+e^x)^2}$$

$$= \frac{2e^{2x}(1+e^x) - e^{2x}e^x}{(1+e^x)^2}$$

(f) $f(x) = \ln\left(x^2 e^x\right) = \ln x^2 + \ln e^x = 2\ln x + x$

$f'(x) = \frac{2}{x} + 1$

2. (10 points) Show that the equation $x^5 + 10x + 3 = 0$ has exactly one real number solution.

Let $f(x) = x^5 + 10x + 3$. Since f is a polynomial, f is continuous and differentiable for $-\infty < x < \infty$. $f(-1) = -8$ and $f(0) = 3$. By the Intermediate Value Theorem, there exists an $-1 < x < 0$ such that $f(x) = 0$. Suppose $f(x)$ has two roots $x_1 < x_2$. Then $f(x_1) = f(x_2) = 0$. By Rolle's Theorem (or the Mean Value Theorem) $f'(c) = 0$ for some $x_1 < c < x_2$. However $f'(x) = 5x^4 + 10 > 0$. Thus $f(x)$ has exactly one real root.

3. Consider the following equation.

$$x^3 + y^3 = 6xy$$

(a) (4 points) Find y'.

$$\frac{d}{dx}(x^3 + y^3) = \frac{d}{dx}(6xy)$$

$$3x^2 + 3y^2 y' = 6y + 6xy'$$

$$x^2 - 2y = (2x - y^2)y'$$

$$y' = \frac{x^2 - 2y}{2x - y^2}$$

(b) (3 points) Find the equation of the tangent line to the graph of the equation at the point with coordinates $(x, y) = (3, 3)$.

slope, $m = \dfrac{3^2 - 2(3)}{2(3) - 3^2} = \dfrac{3}{-3} = -1$

$-1 = \dfrac{y-3}{x-3}$

$-x + 3 = y - 3$

$y = -x + 6$

(c) (3 points) At what points will the tangent lines to the graph of the equation be vertical?

Tangent lines will be vertical when y' is undefined. Thus we need $2x - y^2 = 0$

$y^2 = 2x$

$y = \pm\sqrt{2x}$

$x^3 + \left(\pm (2x)^{1/2}\right)^3 = 6x\left(\pm 2x\right)^{1/2}$

$x^3 \pm 2\sqrt{2}x^{3/2} = \pm 6\sqrt{2}\,x^{3/2}$

$x^3 = \pm 4\sqrt{2}\,x^{3/2}$

$x^6 - 32x^3 = 0 \quad \Rightarrow \quad x = 0$ or $x = 2\sqrt[3]{4}$.

4. (10 points) Use a linear approximation to find the approximate value of $\sqrt{36.1}$.

Let $f(x) = \sqrt{x}$ and $x_0 = 36$.

$L(x) = f(36) + f'(36)(x - 36)$

$= 6 + \frac{1}{12}(x - 36)$

Thus $\sqrt{36.1} \approx L(36.1) = 6 + \frac{1}{12}(36.1 - 36) = 6.008\overline{3}$

5. (10 points) Use Newton's Method to approximate a solution to the equation

$$2x^3 - 6x^2 + 3x + 1 = 0$$

using an initial approximation of $x_0 = 2.5$.

Let $f(x) = 2x^3 - 6x^2 + 3x + 1$

$f'(x) = 6x^2 - 12x + 3$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

n	x_n
0	2.5
1	2.2857
2	2.2288
3	2.2248
4	2.2247