1 Solutions to Wednesday's Review Session

1.1 If $p(x) = \frac{1}{1-x}$, $g(x) = x^3 + 2$, and $h(x) = \frac{1}{x^2}$ simplify the expression $\frac{1}{p(x^2)} - h(g(x))$.

$$\begin{aligned}
\frac{1}{p(x^2)} - h(g(x)) &= \frac{1}{\frac{1}{1-x^2}} - \frac{1}{(x^3+2)^2} \\
&= 1 - x^2 - \frac{1}{(x^3+2)^2} \\
&= \frac{(1-x^2)\cdot(x^3+2)^2 - 1}{(x^3+2)^2}.
\end{aligned}$$

1.2 Find the inverses of the functions $f(x) = \frac{e^x+3}{e^x-2}$ and $g(x) = \frac{3}{2+\ln(x)}$.

$$\begin{gathered}
y = \frac{e^x+3}{e^x-2} \\
\Rightarrow (e^x - 2)y = e^x + 3 \\
\Rightarrow e^x y - 2y = e^x + 3 \\
\Rightarrow e^x y - e^x = 3 + 2y \\
\Rightarrow e^x(y-1) = 3 + 2y \\
\Rightarrow e^x = \frac{3+2y}{y-1} \\
x = \ln\left(\frac{3+2y}{y-1}\right). \\
f^{-1}(x) = \ln\left(\frac{3+2x}{x-1}\right)
\end{gathered}$$

$$\begin{gathered}
y = \frac{3}{2+\ln(x)} \\
\Rightarrow \frac{1}{y} = \frac{2+\ln(x)}{3} \\
\Rightarrow \frac{3}{y} - 2 = \ln(x) \\
\Rightarrow x = e^{\frac{3-2y}{y}} \\
g^{-1}(x) = e^{\frac{3-2x}{x}}.
\end{gathered}$$

1.3 Find a power function that passes through the points $(4, 7)$ and $(7, 8)$.

We know that a power function is any function in the form $f(x) = kx^n$. So, we need to find the constants k and n. We do this by just plugging in the points and solving the resulting equations.

$$7 = k4^n \text{ and } 8 = k7^n$$

$$\Rightarrow \frac{8}{7} = \frac{7^n}{4^n}.$$

$$\Rightarrow \frac{8}{7} = \left(\frac{7}{4}\right)^n$$

$$\Rightarrow \ln\left(\frac{8}{7}\right) / \ln\left(\frac{7}{4}\right) = n$$

$$\rightarrow n = .2386.$$

Now, we just need to find the value of k.

$$7 = k4^{.2386}.$$

$$\Rightarrow \frac{7}{4^{.2386}} = k$$

$$f(x) = 5.0285x^{.2386}.$$

1.4 Express $\sin(4\theta)$ in terms of functions involving $\cos(\theta)$ and $\sin(\theta)$.

$$\sin(4\theta) = \sin(2 \cdot 2\theta) = 2\sin(2\theta)\cos(2\theta) = 4\cos(\theta)\sin(\theta)(\cos^2(\theta) - \sin^2(\theta)).$$

1.5 If $\sin(1) = .84$. Find another angle θ such that $\sin(\theta) = .84$.

$$\pi - 1$$