MATH 124 AND 125

FINAL EXAM REVIEW PACKET ANSWERS

(Revised spring 2008)

1.

t	1.0	1.2	1.4	1.6	1.8
$f'(t)$	1/2	3/4	5/4	7/4	2

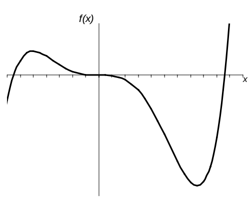
2. __5__ $\lim_{h\to 0}\frac{f(20+h)-f(20)}{h}$ __3__ The slope of f at $x=10$ __1__ $f(16)$

__4__ The average rate of change of f from $x=12$ to $x=24$ __2__ $\left.\frac{dy}{dx}\right|_{x=28}$

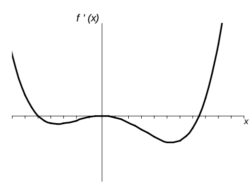
3. a) $g(-2)=3,\ \ g'(-2)=-1$ b) $g(-2)=-3,\ \ g'(-2)=1$

4. a) For $f(150)=125$: If a person weighs 150 pounds, the dose should be 125 milligrams.
For $f'(150)=3$: The dose for a 151 pound person would need to be approximately 3 milligrams higher than a dose for a 150 pound person.
b) 140 milligrams.

5. Graph of $f(x)$

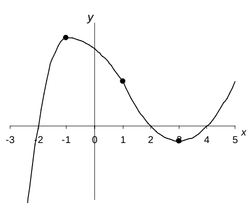


Graph of $f'(x)$



6. a) False b) True c) False d) False

7.



8. a) $\frac{1}{2}$ b) $\frac{1}{1+e}$ c) 0 d) 1 e) Does not exist.

9. $\lim_{h\to 0}\frac{e^{2(3+h)}-e^{2(3)}}{h}=f'(3)$ where $f(x)=e^{2x}$. $f'(3)=2e^{6}$.

10. $V(t)=25000(.85)^t$, $V'(3)=25000\ln(.85)(.85)^3\approx -2495.17$ dollars per year.

11. a) $x=-2$, $x=2$, $x=4$ b) $x=-1$, $x=3$ c) $x=1$

12. a) $(-\infty,-2)$, $(2,4)$ b) $(-1,3)$

13. a) $\dfrac{dy}{dx}=\dfrac{1}{1+(a+x)^2}$ b) $\dfrac{dy}{dx}=\dfrac{-ax}{\left(a^2+x^2\right)^{3/2}}$ c) $\dfrac{dy}{dx}=-3a\cos^2(ax)\sin(ax)$

d) $\dfrac{dy}{dx}=-\ln a\left(a^{-x}\right)+ax^{a-1}$ e) $\dfrac{dy}{dx}=\dfrac{1}{a^2}\cosh\left(\dfrac{x}{a}\right)$

14. a) $y=10x-27$ $h'(x)=2f'(x)$ b) Decreasing $g'(4)=-\dfrac{56}{9}$ $g'(x)=\dfrac{f(x)2x-x^2f'(x)}{\left(f(x)\right)^2}$

c) $k'(2)=20$ $k'(x)=f'(x^2)2x$ d) $m'(4)=-5e^{-3}$ $m'(x)=e^{-f(x)}(-1)f'(x)$

15. $x=5$, $x=-1$

16. a) $\dfrac{dm}{dv}=\dfrac{m_o v}{c^2\left(1-\left(v^2/c^2\right)\right)^{3/2}}$ b) $g'(x)=\begin{cases}2x & x<-3\\ -2x & -3<x<3\\ 2x & x>3\end{cases}$

Note: $g'(x)$ is not defined at $x=3$ and $x=-3$.

17. a) continuous, but not differentiable b) continuous and differentiable

c) $f'(x)=\begin{cases}2 & x<-1\\ 2x & -1<x\le 2\\ 4 & x>2\end{cases}$

18. $f(h)=\dfrac{4}{5}h+20$

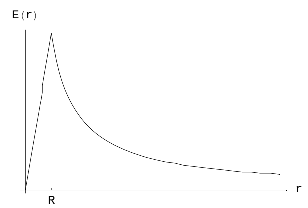
19. $k=15$

20. a) $\dfrac{3}{4}$ $2xy+x^2\dfrac{dy}{dx}+2\pi\sin\left(\pi x\right)=\dfrac{1}{x}-3y^2\dfrac{dy}{dx}$ b) $y=\dfrac{3}{4}x-\dfrac{7}{4}$

21. a) Yes. $E(R) = \lim_{r \to R^+} E(r) = kR$

b) No. The slope is *k* for $r < R$, but the slope approaches –*k* as *r* approaches *R* from the right.

c)



d)

$$\frac{dE}{dr} = \begin{cases} k & r < R \\ \dfrac{-kR^2}{r^2} & r > R \end{cases}$$

22. a) $t = 3$ is a local minimum, $(3, -1/27)$
b) $t = 4$ is the inflection point, $(4, -1/32)$
c) $t = 2$ is the global maximum, $t = 3$ is the global minimum

23. a) $\left(-a, 2a^4 + 2a^3\right)$ is the local maximum, $\left(a, 2a^4 - 2a^3\right)$ is the local minimum.
b) $\left(0, 2a^4\right)$ is the inflection point.

24. a) $\lim_{t \to \pi} \dfrac{t^2 - \pi^2}{\sin t} = -2\pi$ Use L'Hopital's rule.
b) $\lim_{\theta \to 0} \dfrac{\sin(2\theta)}{\sin(7\theta)} = \dfrac{2}{7}$ Use L'Hopital's rule.
c) $\lim_{x \to \infty} \arctan x = \pi/2$

25. $A = 1/16$, $B = 1/2$ Use $f(4) = 1$ and $f'(4) = 0$ to find *A* and *B*.

26. a) $\left(e^a, 0\right)$ b) $t = e^{a-1}$ is a local maximum.

27. $a = 0$, $b = 4$, $k = 3$

28. a) $c = -15$ b) $c = 200$

29. a) $t = 2$, $(-3, -16)$ Time when $\dfrac{dx}{dt} = 0$ and $\dfrac{dy}{dt} = 0$ b) $t = 6$ Time when $y = 0$.

30. The maximum volume is $\dfrac{L^3}{1728}$ cubic inches. The dimensions are $\dfrac{L}{12} \times \dfrac{L}{12} \times \dfrac{L}{12}$.

$$V(x) = x \cdot x \cdot \left(\frac{L - 8x}{4}\right)$$

31. The width is $\frac{10}{\sqrt{3}}$ and the height is $\frac{50}{3}$. $A(x)=(2x)\cdot(25-x^2)$

32. The minimum cost is $(9.6)V^{2/3}$ dollars. The dimensions are $(0.8)V^{1/3}$ by $(0.8)V^{1/3}$ by $\frac{V^{1/3}}{0.64}$.

$C(x)=5x^2+\frac{5.12V}{x}$

33. The wavelength is $w=c$ $V'(w)=\frac{k}{2}\left(\frac{w}{c}+\frac{c}{w}\right)^{-1/2}\left(\frac{1}{c}-\frac{c}{w^2}\right)$

34. a) $f'(x)$ is the graph that looks linear.
b) $x=-1$ is a local minimum. $x=5$ is a local maximum.
$g'(x)=-f(x)e^{-x}+e^{-x}f'(x)$, $g'(x)=0$ when $f(x)=f'(x)$.

35. a) 25 miles per gallon during the first 70 miles. $50/3\approx 16.67$ miles per gallon in the next 30 miles.
b) The total gallons used t hours into the trip. $k(0.5)=1.4$ gallons.
c) $k'(0.5)=2.8$, $k'(1.5)=1.8$ gallons per hour. Gas consumption is better in the first 70 miles, but gas is being consumed more quickly.

36. The camera is rotating at $1/24$ radians per minute. $x=\tan\theta$, $\frac{d\theta}{dt}=-2.5$

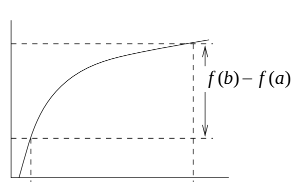
37. The height is growing at $\frac{5}{432\pi}$ feet per minute. $V=\frac{1}{3}\pi(3h)^2h$.

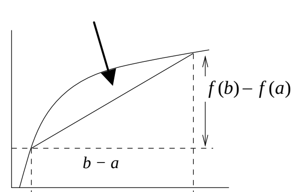
38. The resistance is decreasing at 10 ohms per minute. $\frac{dV}{dt}=I\frac{dR}{dt}+R\frac{dI}{dt}$

39. a) The lower estimate is 0.28. The upper estimate is 0.48.
b) Using the Fundamental Theorem of Calculus, $\int_{1.2}^{1.6} f'(t)dt = f(1.6)-f(1.2)=0.5$.

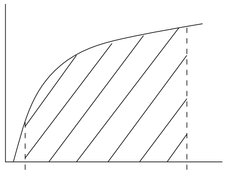
40. a) Object B b) Objects A and D c) Objects A and D d) Object C

41. a) length b) slope of the line

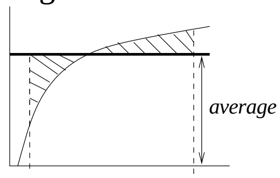




c) area



d) length



42. __1__ $\sum_{k=1}^{10} g(t_k)\Delta t$ __3__ $\sum_{k=0}^{9} g(t_k)\Delta t$ __2__ $\lim_{n\to\infty}\sum_{k=1}^{n} g(t_k)\Delta t$

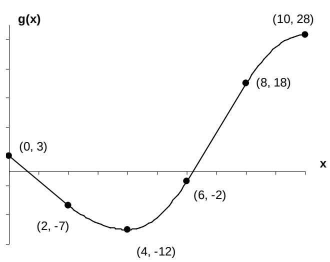
43. a) $\frac{b}{3}x^3 + x + c$ b) $b\ln|x| + \frac{1}{2}x^2 + c$ c) $\frac{1}{2}\ln\left|b + x^2\right| + c$ d) $\frac{1}{b}\arctan(bx) + c$

44. a) $\int_0^4 x(4-x)dx = \frac{32}{3}$ b) $\int_0^4 \left((x+2) - (x^2 - 3x + 2)\right)dx = \frac{32}{3}$

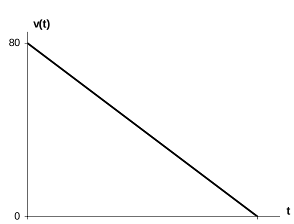
45. 2400 people will be added to the city. $\int_0^4 \left(3\sqrt{t} + 2\right)dt = 24$

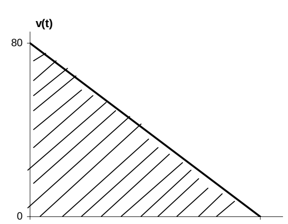
46. $r(t) = \begin{cases} 75 & 0 \le t < 2 \\ -3(t-2)^2 + 75 & 2 \le t \le 7 \end{cases}$ $\int_0^2 75dt + \int_2^7 \left(-3(t-2)^2 + 75\right)dt = 400$ gallons

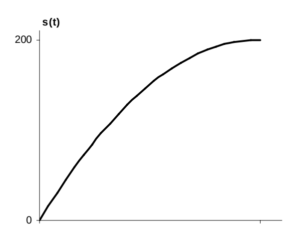
47.



48. a) $V(t) = -16t + 80$ b) 200 feet c) $s(t) = -8t^2 + 80t$







49. a) $F(0)=\int_0^0 e^{-t^2}dt=0$

b) $F'(x)=\dfrac{d}{dx}\int_0^x e^{-t^2}dt=e^{-x^2}$

c) Increasing.

d) Concave down. $F''(x)=-2xe^{-x^2}$

50. $\dfrac{1}{\pi/4-0}\int_0^{\pi/4}\dfrac{3}{\cos^2 x}dx=\dfrac{12}{\pi}$

51. a) $\int_{-\pi}^{\pi}\ln\left(5+4\cos x\right)dx=4\pi\ln 2$ $f(x)$ is an even function.

b) $\int_0^{\pi/2}\ln\left(5+4\cos(2x)\right)dx=\pi\ln 2$ Let $u=2x$ and change the endpoints.

52. a) True b) True c) False d) True e) False

53. a) $17/6$ b) 20 c) $37/2$

54. a) $g(C)>g(D)$ b) $g'(B)>g'(C)$ c) $g''(A)>g''(B)$