APMA: 0200
Homework #5

Due Date: October 30, 2015

In this homework we will be further analyzing the mathematical model of love R (J) that Romeo (Juliet) feels for Juliet (Romeo). The model is the following:

$$\frac{dR}{dt} = aJ + mR + r$$

$$\frac{dJ}{dt} = bR + nJ + s$$

where a, b, m, n, r, s are constants that can have any sign. The questions are very open ended, you can use any tools to try to analyze them. In some cases the trick $\frac{dR}{dJ}$ might be useful in others it might be possible to write down a related differential equation by considering the total love: $L = R + J$. In all cases draw a phase portrait for the behavior of the couple as a function of time.

1. Consider the affair described by

$$\frac{dR}{dt} = J$$

$$\frac{dJ}{dt} = -R + J$$

    (a) Characterize the romantic styles of Romeo and Juliet.

    (b) By drawing a phase portrait for this system, classify the stability of any fixed points. What does this imply for the affair?

2. (Out of touch with their own feelings) Suppose Romeo and Juliet react to each other, but not to themselves:

$$\frac{dR}{dt} = aJ,$$

$$\frac{dJ}{dt} = bR.$$

    What happens in the long run for all possible signs of a and b?

3. (Fire and Water) Do opposites attract? Analyze the following system

$$\frac{dR}{dt} = aR + bJ,$$

$$\frac{dJ}{dt} = -bR - aJ,$$

    for all possible signs of a and b.

4. (Peas in a Pod) If Romeo and Juliet are romantic clones

$$\frac{dR}{dt} = aR + bJ,$$
$$\frac{dJ}{dt} = bR + aJ,$$

should they expect boredom or bliss?

5. (Romeo the Robot) Nothing could ever change the way Romeo feels about Juliet:

$$\frac{dR}{dt} = 0,$$
$$\frac{dJ}{dt} = aR + bJ.$$

Does Juliet end up loving him or hating him.