Homework #4.

#5.1.10

For each of the following systems determine if the system is attracting, Liapunov stable, asymptotically stable or none of the above.

Solution:

a.) $\dot{x} = y$ $\rightarrow$ Liapunov stable
$\dot{y} = -4x$

b.) $\dot{x} = 2y$ $\rightarrow$ None of the above
$\dot{y} = x$

c.) $\dot{x} = 0$ $\rightarrow$ None of the above
$\dot{y} = x$

d.) $\dot{x} = 0$ $\rightarrow$ Liapunov stable.
$\dot{y} = -y$

e.) $\dot{x} = -x$ $\rightarrow$ Asymptotically stable.
$\dot{y} = -5y$

f.) $\dot{x} = x$ $\rightarrow$ None of the above.
$\dot{y} = y$

#5.2.4

Sketch a phase portrait for

$$\begin{cases} \dot{x} = 5x + 10y \\ \dot{y} = -x - y \end{cases}$$

Solution:

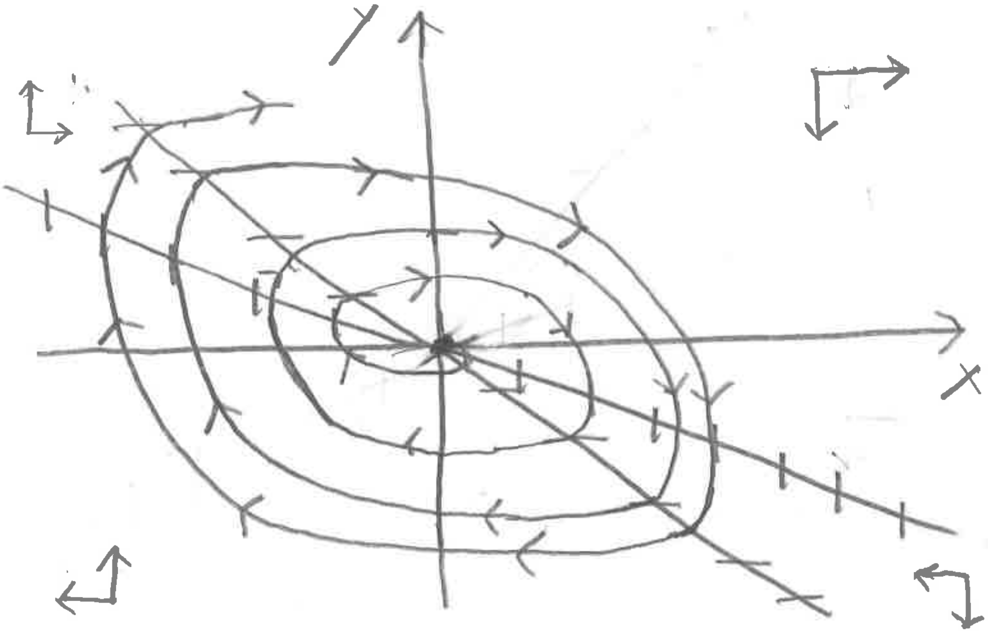
The matrix for this system is

$$A = \begin{pmatrix} 5 & 10 \\ -1 & -1 \end{pmatrix}$$

with eigenvalues

$$\lambda_{1,2} = \frac{4 \pm \sqrt{16-20}}{2} = 2 \pm i$$

The fixed points are unstable spirals.



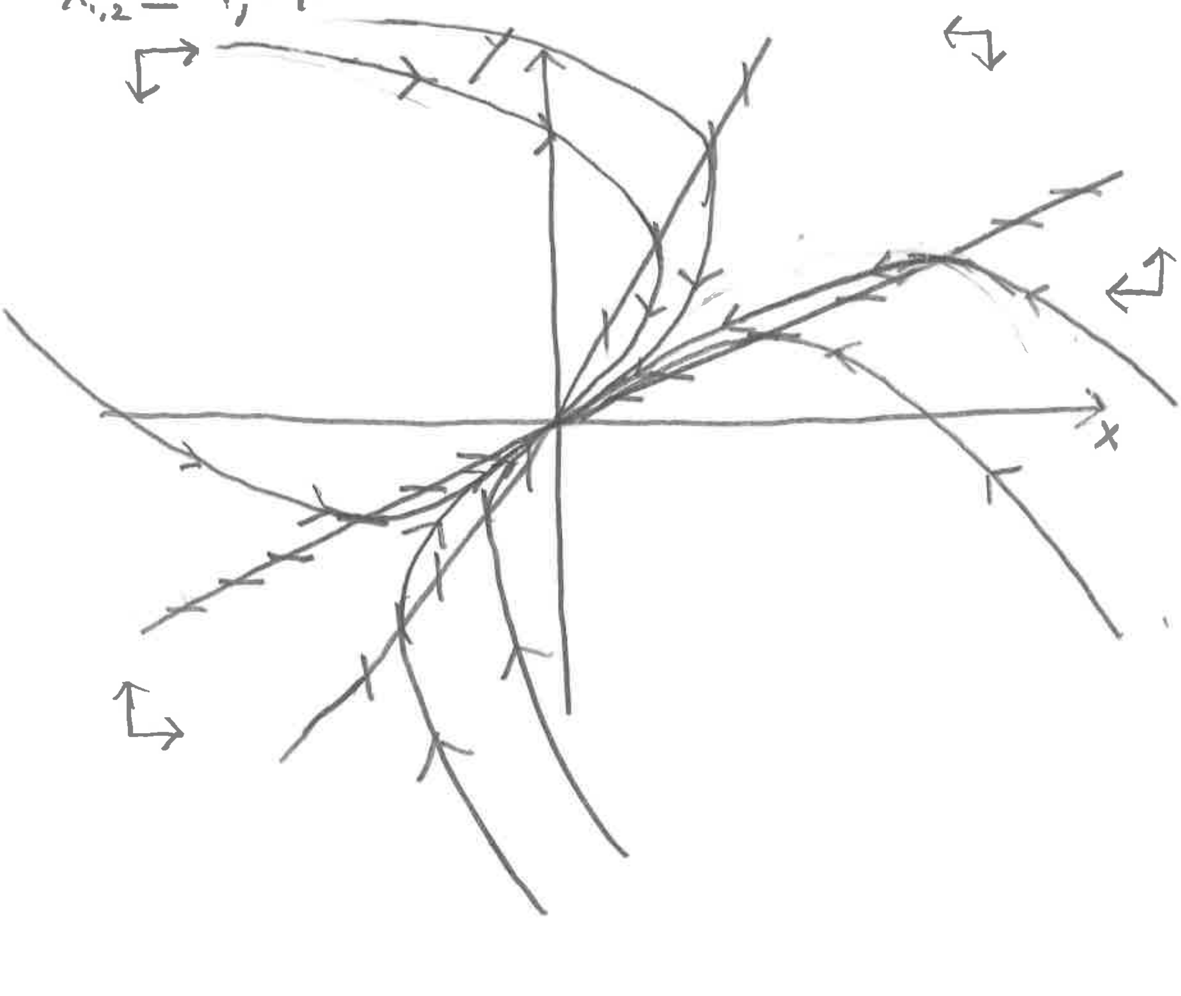
#5.2.6.

$$\begin{cases} \dot{x} = -3x+2y \\ \dot{y} = x-2y \end{cases}$$

Solution:

The eigenvalues for this system are

$\lambda_{1,2} = -1, -4$



#5.2.8

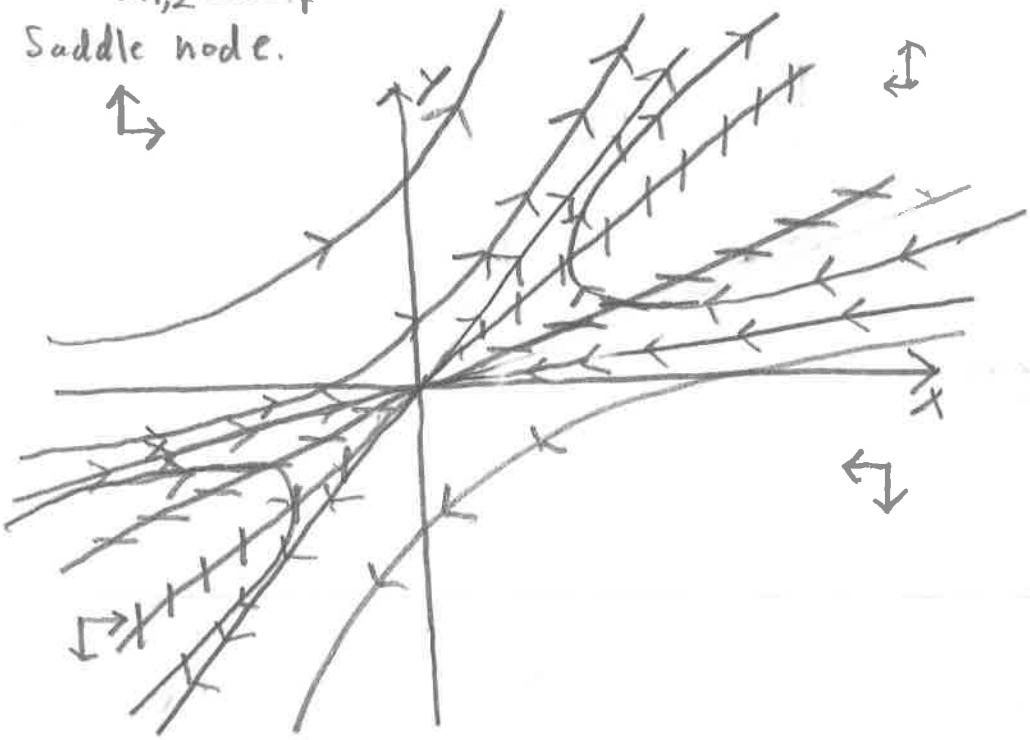
$$\begin{cases} \dot{x} = -3x + 4y \\ \dot{y} = -2x + 3y \end{cases}$$

Solution:

The eigenvalues are

$$\lambda_{1,2} = \pm 1$$

Saddle node.



The eigenvectors are

$$\vec{\lambda}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \vec{\lambda}_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

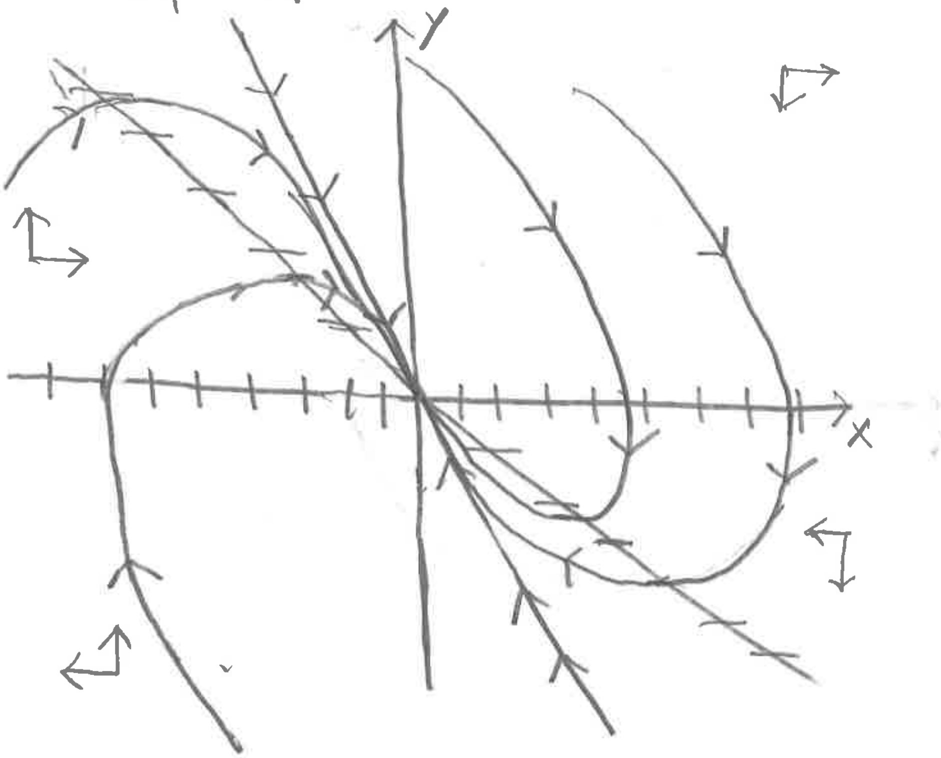
5.2.10.

$$\begin{cases} \dot{x} = y \\ \dot{y} = -x - 2y \end{cases}$$

Solution:

The eigenvalues are

$\lambda_1 = -1$



eigenvector $\vec{\lambda}_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$