AM 33, Fall 2001, Third Practice Exam

1. Let

$$f(t) = \begin{cases} 2 & ; \quad 0 < t < 2 \\ t & ; \quad t \geq 2 \end{cases} .$$

Compute the Laplace transform of f. Solve the IVP

$$y'(t) + 2y(t) = f(t), \qquad y(0) = 1.$$

2. Solve the following integro-differential equation by taking Laplace transform

$$y'(t) = 2y(t) + 2e^{3t} + 2\int_0^t e^{3(t-\tau)} y(\tau) d\tau, \qquad y(0) = 1.$$

3. Using Laplace transform method to solve the following system of equations:

$$\begin{aligned} \frac{dz_1}{dt} + \frac{dz_2}{dt} &= 1 - 4z_1 \\ \frac{dz_1}{dt} &= 2z_1 - z_2 + t^2 \end{aligned}$$

with initial condition $z_1(0) = 2, z_2(0) = -1$. Note: the computation here is a bit messy.

4. Consider the differential equation

$$ty'' + y' + ty = 0; \qquad t \geq 0,$$

with initial condition $y(0) = 1$, $y'(0) = 0$.

(a) Let $Y = \mathcal{L}\{y\}$. Argue that Y satisfies the differential equation

$$(s^2 + 1)\frac{dY}{ds} + sY = 0.$$

(b) Solve the above equation to conclude that

$$Y(s) = \frac{C}{\sqrt{1 + s^2}}$$

for some constant C.

5. An auto insurance company classifies its policyholders as "good risks", "bad risks" or "average risks": 30% are deemed "good risks", 20% are deemed "bad risks", and 50% are deemed "average risks". Historical data suggest that 5% of the "good risks", 40% of the "bad risks", and 10% of the "average risks" will be involved in an accident in the coming year.

(a) What is the probability that a randomly chosen policyholder will involve in an accident in the coming year?

(b) An accident claim has just been filed with the company. What is the probability that this policyholder was classified as a "good risk"? A "bad risk"? An "average risk"?