AM 33, Fall 2001, Second Practice Exam

1. Find the general solution to the differential equation

$$y'' + y' - 6y = e^{2x} - 2.$$

2. One solution of

$$x^2 y^{''} - 2xy^{'} + (2 - 9x^2)y = 0$$

   is $y_1(x) = xe^{3x}$.

   - Find a second linearly independent solution y_2.
   - Calculate the Wronskian $W(y_1, y_2)(x)$.
   - Solve the initial value problem for the given ODE and data $y(1) = 1, y^{'}(1) = 0$.

3. Consider the second order linear equation

$$y'' + p(t)y' + q(t)y = 0, \qquad t \geq 0,$$

   where $p(t)$ and $q(t)$ are continuous functions. Suppose that $y_1(t)$ satisfies the equation together with the data $y_1(1) = 1, y_1'(1) = 3$, and that $y_2(t)$ satisfies the equation together with the data $y_2(1) = 2, y_2'(1) = 6$. Are the function y_1 and y_2 linearly independent? Why? Let $\bar{y}_1 = y_1 + y_2$ and $\bar{y}_2 = y_1 - y_2$. Are $\bar{y}_1$ and $\bar{y}_2$ linearly independent? Why?

4. The homogeneous equation

$$2t^2 y'' + 3ty' - y = 0, \qquad t > 0$$

   has two solutions $y_1(t) = \sqrt{t}$, $y_2(t) = \frac{1}{t}$. Use the method of variation of parameters to find the general solution of the non-homogeneous equation

$$2t^2 y'' + 3ty' - y = \frac{1}{t}, \qquad t > 0.$$