AM 33, Fall 2001, Practice Exam

1. Find all critical points of the differential equation

$$\frac{dy}{dt} = 4y - y^3.$$

   Classify each critical point as stable, unstable or semistable.

   Let $y(t)$ be the solution to this differential equation with $y(0) = 1$. Without solving the differential equations, state whether $y(t)$ is increasing or decreasing as t increases, and find them limit of $y(t)$ as $t \to \infty$.

2. Solve the initial value problem

$$y'(x) = \frac{-(x + y(x))^{1/2}}{(x + y(x))^{1/2} + 2}, y(0) = 1.$$

   This ODE is not easily solved by the techniques introduced so far. You may want to consider a change of variable (as in the case of Bernoulli and homogeneous equations) that will make the problem simpler.

3. Consider the differential equation

$$(y(x) + e^x) + \left(e^{3y(x)-x} + 1\right) y'(x) = 0.$$

   (a) Find an integrating factor of the form $\mu = \mu(x)$ or $\mu = \mu(y)$ that turns this equation into an exact equation.

   (b) Find an implicit relationship that solutions of this ODE satisfy.

4. Consider the ODE

$$y'(x) = \frac{x}{1 + y(x)}.$$

   (a) Is this equation linear?

   (b) Identify the set of points in the (x, y)-plane where the existence and uniqueness theorem *fails* to hold.

   (c) Solve the IVP given by this ODE and the condition $y(1) = 1$.