AM 33: Homework # 10

Due date: Dec 3 for students in section 1 (E) and Dec 4 for students in section 2 (J).

The book we refer to is Boyce & DiPrima, *Elementary Differential Equations and Boundary Value Problems* (7th Edition).

- Section 6.3, problems 9, 32.
- Section 6.4, solve the initial value problems 9, 10. Note: no need to draw the graphs.
- Section 6.5, problems 6 (no need to draw the graph), 25.
- Section 6.6, problems 3, 16, 21.
- Suppose that $f(t)$ is a continuous function for $t \geq 0$.
  1. Show that $(f * 1)(t) = \int_0^t f(s)\, ds$.
  2. Show that
  $$\int_0^t \int_0^{t_1} \int_0^{t_2} \cdots \int_0^{t_{n-2}} \int_0^{t_{n-1}} f(t_n) dt_n\, dt_{n-1} \cdots dt_3\, dt_2\, dt_1 = (f * 1 * 1 * \cdots * 1)(t);$$
  here in the above convolution 1 is convoluted n times.
  3. Use Laplace transform method to prove the formula
  $$\int_0^t \int_0^{t_1} \int_0^{t_2} \cdots \int_0^{t_{n-2}} \int_0^{t_{n-1}} f(t_n) dt_n\, dt_{n-1} \cdots dt_3\, dt_2\, dt_1 = \frac{1}{(n-1)!} \int_0^t (t-s)^{n-1} f(s)\, ds$$