Solution 5

13.9 To test $H_0: \mu_1 = \mu_2 = \mu_3$ VS. H_a: one or more of μ_i's differ.

$y_{1\cdot} = 14(.93) = 13.02$, $y_{2\cdot} = 14(1.21) = 16.94$, $y_{3\cdot} = 14(.92) = 12.88$

$CM = \frac{1}{n}(y_{1\cdot} + y_{2\cdot} + y_{3\cdot})^2 = \frac{1}{42}(42.84)^2 = 43.6968$

$\Rightarrow SST = \frac{1}{14}(y_{1\cdot}^2 + y_{2\cdot}^2 + y_{3\cdot}^2) - CM = .7588$

$\Rightarrow MST = SST/k-1 = .7588/2 = \boxed{.3794}$

$s_1^2 = 14(.04)^2 = .0224$, $s_2^2 = 14(.03)^2 = .0126$, $s_3^2 = 14(.04)^2 = .0224$

$\Rightarrow SSE = \sum_{i=1}^{3}(n_i - 1)s_i^2 = 13(.0224 + .0126 + .0224) = .7462$

$\Rightarrow MSE = SSE/n-k = .7462/39 = \boxed{.019133}$

$\Rightarrow F = MST/MSE = \boxed{19.83}$

The RR with df. 2 & 39 is $F > F_{2,39}(.05) = 3.23$.

The null hypothesis is rejected & there is a difference between the means.

p-value $< .005$.

13.13 We have a completely randomized design with four treatments.

$CM = \frac{1}{n}\left(\sum_i\sum_j y_{ij}\right)^2 = \frac{1}{19}(110.6)^2 = 643.8084$

Total $SS = \sum_i\sum_j y_{ij}^2 - CM = 652.26 - 643.8084 = 8.4516$

$SST = \sum_i \frac{y_{i\cdot}^2}{n_i} - CM = \frac{(30.4)^2}{5} + \frac{(32.2)^2}{5} + \frac{(23.9)^2}{5} + \frac{(24.1)^2}{4} - 643.8084$

$= 7.8361 \quad \Rightarrow \quad MST = SST/k-1 = 7.8361/3 = \boxed{2.61203}$

$SSE = \text{Total } SS - SST = .6155 \Rightarrow MSE = SSE/n-k = .6155/15 = \boxed{.04103}$

$\Rightarrow F = MST/MSE = \boxed{63.66}$

The critical value for F with $\alpha = .005$ and df. $= 3$ & 15 is $F = 6.48 < 63.66$. Thus, p-value $< .005$

We conclude that there is a difference in mean dissolved oxygen content for the four locations.

13.21 Need to consider $(\overline{Y}_{1\cdot} - \overline{Y}_{2\cdot}) \pm t_{.025,39}\, s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$

where $s = \sqrt{MSE} = .1383$

95% CI is

$$(.93 - 1.21) \pm 1.96(.1383)\sqrt{\frac{1}{14}+\frac{1}{14}}$$

$$= -.28 \pm .102 = (-.382, -.178)$$

At the 95% confidence level, we would conclude that there is a significant difference between the mean bone densities for the two groups of women since the confidence interval formed contains all negative values. This suggests that $\mu_2 > \mu_1$.

13.30 The estimator for $\frac{\mu_1+\mu_2}{2} - \mu_4$ is $\frac{\overline{y}_{1\cdot}+\overline{y}_{2\cdot}}{2} - \overline{y}_{4\cdot}$, which has variance $\frac{1}{4}\left(\frac{\sigma^2}{n_1}+\frac{\sigma^2}{n_2}\right)+\frac{\sigma^2}{n_4}$

The 95% CI is

$$\left(\frac{\overline{y}_{1\cdot}+\overline{y}_{2\cdot}}{2} - \overline{y}_{4\cdot}\right) \pm t_{.025,15}\sqrt{s^2\left(\frac{1}{4n_1}+\frac{1}{4n_2}+\frac{1}{n_4}\right)}$$

$$= (6.26 - 6.025) \pm 2.131\sqrt{.04103\left(\frac{1}{20}+\frac{1}{20}+\frac{1}{4}\right)}$$

$$= .235 \pm .255 = (-.020, .490)$$

13.62

(a) The complete model is $Y_{ij} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon$

where $x_1 = \begin{cases} 1 & \text{if method A} \\ 0 & \text{otherwise} \end{cases}$ and $x_2 = \begin{cases} 1 & \text{if method B} \\ 0 & \text{otherwise} \end{cases}$.

Then for the complete model,

$$Y = \begin{pmatrix} 73 \\ 83 \\ 76 \\ 68 \\ 80 \\ 54 \\ 74 \\ 71 \\ 79 \\ 95 \\ 87 \end{pmatrix}, X = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, X'X = \begin{pmatrix} 11 & 5 & 3 \\ 5 & 5 & 0 \\ 3 & 0 & 3 \end{pmatrix}, (X'X)^{-1} = \begin{pmatrix} \frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{8}{15} & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} & \frac{2}{3} \end{pmatrix},$$

$$\hat{\beta} = (X'X)^{-1} X'Y = \begin{pmatrix} 87 \\ -11 \\ -20.67 \end{pmatrix}.$$

$SSE_C = Y'Y - \hat{\beta} X'Y = 65,286 - 54,787.33 = 498.67$ with d.f.=11-3=8.

The reduced model is $Y_{ij} = \beta_0 + \varepsilon$ and the X matrix becomes

$$X = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, X'X = 11, (X'X)^{-1} = \frac{1}{11}, \hat{\beta} = 76.3636.$$

$SSE_R = 65,286 - 64,145.455 = 1140.5455$ with d.f.= 11-1 = 10.

Then $s_2^2 = \frac{498.67}{8} = 62.3333$ and $s_3^2 = \frac{SSE_R - SSE_C}{10-8} = \frac{641.8788}{2} = 320.9394$ and $F = \frac{s_3^2}{s_2^2} = 5.15$, which is compared to $F_{.05} = 4.46$ with d.f. of 2 and 8. There is evidence of a difference in treatment means.

(b) The hypothesis to be tested is $H_0 : \mu_A - \mu_C = 0$ VS. $H_a : \mu_A - \mu_C \neq 0$. With s_2^2 used to estimate σ^2, the test statistics is $\frac{\bar{y}_A - \bar{y}_C}{\sqrt{MSE_2\left(\frac{1}{5}+\frac{1}{3}\right)}} = \frac{76-87}{\sqrt{62.333\left(\frac{8}{15}\right)}} = -1.91$.

The RR is $|t| > t_{.025,8} = 2.306$, and the null hypothesis is not rejected. There is not significant evidence of a difference between A and C.

15.2 a. A sign test is employed to test the hypothesis

$H_0: p = \frac{1}{2}$ VS $H_a: p \neq \frac{1}{2}$

where $p = P$ (school A exceeds school B in test score)

Use test statistic M = # of times school A exceeds school B in test score

Twin Pair	sign (A−B)
1	+
2	+
3	−
4	+
5	+
6	−
7	+
8	+
9	−
10	+

$\Rightarrow$ observed value of M is $m = 7$

$$\text{p-value} = P(M \geq 7 \text{ or } M \leq 3) = 2P(M \geq 7)$$
$$= 2\left[\binom{10}{7}\left(\tfrac{1}{2}\right)^{10} + \binom{10}{8}\left(\tfrac{1}{2}\right)^{10} + \binom{10}{9}\left(\tfrac{1}{2}\right)^{10} + \binom{10}{10}\left(\tfrac{1}{2}\right)^{10}\right]$$
$$= .344 > .05$$

So, we do not reject H_0

b. Test the hypothesis $H_0: p = \frac{1}{2}$ VS $p > \frac{1}{2}$

$\text{p-value} = P(M \geq 7) = .172 > .05$

Therefore, we still fail to reject H_0

15.3 Let $p = P$(judge favors mixture B)

M = # of judges favouring mixture B

The hypothesis to be tested is $H_0: p = \frac{1}{2}$ vs $H_a: p \neq \frac{1}{2}$

With $n = 10$, the observed value of M is $m = 2$.

p-value $= 2P(M \leq 2) = 2(.055) = .11 > .05$

So, we conclude that there is no significant difference between the taste of A & B.

15.18 (a) The data with their corresponding ranks are given in the table at the right.

Notice the observations corresponding to rank 9 & 10 and 13 & 14 were tied.

Then, the two possible values for U are $U_A = n_1 n_2 + \frac{n_1(n_1+1)}{2} - W_A$

$= 126 - 94 = 32$

$U_B = 126 - 77 = 49$

Thus, $U = 32$

	A	B
	1	16
	17	8
	12	11
	13.5	2
	5	4
	3	7
	18	9.5
	9.5	6
	15	13.5
Rank Sum	94	77

Referring to Table 8 & indexing $n_1 = n_2 = 9$,

p-value $= 2P(U \leq 32) = 2(.2447) = .4894$ is relatively large.

Therefore we fail to reject H_0 and conclude that there is no significant difference in locations of brightness measurements for the two processes.

15.19 If H_a is true, we would expect that the batteries from plant A to fail later than those from plant B. Hence the observations from plant A will be ranked near the end of the sequence & U-statistic (= # of observations from plant A that precede each observation from plant B) will be small. Hence, small values of U will tend to contradict H_0, and a lower-tailed RR is desired.

Recall that when H_0 is true, $E(U) = \frac{n_1 n_2}{2}$ &

$V(U) = n_1 n_2 (n_1 + n_2 + 1)/12$

Also, for large n_1 & n_2, $z = \frac{U - E(U)}{\sigma_U}$ is approximately

$N(0,1)$. The null hypothesis will be rejected if $z < -1.645$

Here, $n_1 = n_2 = 15$, $W_A = 276$, $W_B = 189$

$\Rightarrow U_A = n_1 n_2 + \frac{n_1(n_1+1)}{2} - W_A = 345 - 276 = 69$

Also, $E(U) = \frac{n_1 n_2}{2} = \frac{225}{2} = 112.5$

& $V(U) = n_1 n_2 (n_1 + n_2 + 1)/12 = 15 \times 15 \times 31 / 12 = 581.25$

$\Rightarrow z = \frac{69 - 112.5}{\sqrt{581.25}} = -1.80 < -1.645$

So, we reject H_0. There is sufficient evidence to conclude that the length of life for the experimental batteries tend to be greater than the length of life for the standard batteries at significant level .05.

14.1 Let p_i = probability that a car will be driven in Lane i $(i = 1, 2, 3, 4)$

H_0: $p_1 = p_2 = p_3 = p_4$

$$X^2 = \sum_{i=1}^{4} \frac{[n_i - E(n_i)]^2}{E(n_i)}$$

$$= \frac{(294-250)^2}{250} + \frac{(276-250)^2}{250} + \frac{(238-250)^2}{250} + \frac{(192-250)^2}{250}$$

$$= \boxed{24.48} \quad \text{with d.f.} = 4-1=3$$

$\chi^2_{3, 0.005} = 7.81 < 24.48$.

So, reject H_0

p-value < .005.

14.8 Calculate $\hat{\lambda} = \bar{y} = \frac{0(296)+1(74)+\cdots+8(1)}{414} = \boxed{.4831}$

The observed and estimated cell counts are:

y	n_i	$\hat{p}_i$	$\hat{E}(n_i)$
0	296	.6169	255.38
1	74	.298	123.38
2	26	.072	29.80
≥3	18	.0131	5.44

Then $X^2 = \frac{(296-255.38)^2}{255.38} + \cdots + \frac{(18-5.44)^2}{5.44} = \boxed{55.71}$ with d.f. $k-2=2$.

$\chi^2_{2, .05} = 5.99 < 55.71$ So, reject H_0. The data do not come from a Poisson dist.