**11.61**

If the minimum value is to occur at $x_0 = 1$, then this implies that $\beta_1 + 2\beta_2 = 0$.

To test the claim, let $\alpha = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$, so that $U = \alpha'\hat{\beta} = \hat{\beta}_1 + 2\hat{\beta}_2$, and the hypothesis to be tested is $H_0 : E(U) = 0$ VS. $H_a : E(U) \neq 0$.

From exercise 11.56 in last homework, we have $\hat{\beta} = \begin{bmatrix} -.7143 \\ -.1429 \\ .1429 \end{bmatrix}$,

$$(X'X)^{-1} = \begin{bmatrix} .3333 & 0 & -.0476 \\ 0 & .0357 & 0 \\ -.0476 & 0 & .0119 \end{bmatrix}, \text{ and } s^2 = .14286.$$

Then $\alpha'(X'X)^{-1}\alpha = .083334$, and $U = \hat{\beta}_1 + 2\hat{\beta}_2 = .142861$. The test statistic is

$$t = \frac{U - E(U)}{\sqrt{s^2\left(\alpha'(X'X)^{-1}\alpha\right)}} = \frac{.142861}{\sqrt{(.14286)(.08333)}} = 1.31.$$

The RR with $\alpha = .05$ is $|t| > 2.776$ (d.f.=4), and $H_0 : E(U) = 0$ is not rejected.

**11.62**

(b) In MATLAB, define

$$Y = \begin{pmatrix} 22.2 \\ 19.4 \\ 22.1 \\ 14.2 \\ 24.5 \\ 24.1 \\ 19.6 \\ 12.7 \\ 24.4 \\ 25.2 \\ 23.5 \\ 19.3 \\ 25.9 \\ 28.4 \\ 16.5 \\ 16.0 \end{pmatrix}, X = \begin{pmatrix} 1 & -1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 & 1 \\ 1 & -1 & 1 & 1 & 1 \\ 1 & 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 \\ 1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & -1 \\ 1 & -1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 & -1 \end{pmatrix} \Rightarrow X'Y = \begin{pmatrix} 338 \\ -50.2 \\ -19.4 \\ -2.6 \\ -20.4 \end{pmatrix}, (X'X)^{-1} = \begin{pmatrix} \frac{1}{16} & & & & \\ & \frac{1}{16} & & & \\ & & \frac{1}{16} & & \\ & & & \frac{1}{16} & \\ & & & & \frac{1}{16} \end{pmatrix},$$

$$\hat{\beta} = (X'X)^{-1} X'Y = \begin{pmatrix} 21.125 \\ -3.1375 \\ -1.2125 \\ -.1625 \\ -1.275 \end{pmatrix}$$

and the fitted model is: h $\hat{y} = 21.125 - 3.1375x_1 - 1.2125x_2 - .1625x_3 - 1.275x_4$.

(c)

Calculate $SSE = Y'Y - \hat{\beta}'X'Y = 98.8125$, and $s^2 = \frac{SSE}{n-(k+1)} = \frac{98.8125}{16-5} = 8.98$.

The hypothesis to be tested is

$H_0 : \beta_i = 0$ VS. $H_a : \beta_i \neq 0$ for i=1,2,3,4.

Each will be based on the test statistic $t_i = \frac{\hat{\beta}_i}{s\sqrt{c_{ii}}} = \frac{4\hat{\beta}_i}{\sqrt{8.98}} = \frac{\hat{\beta}_i}{.7493} \sim t_{11}$, i.e.

$t_1 = \frac{-3.1375}{.7493} = -4.19, t_2 = -1.62, t_3 = -.22, t_4 = -1.70$. (d.f.=11)

Refer to the tables, the corresponding p-values are:

$p_1 < .005; .05 < p_2 < .1; p_3 > .1; .05 < p_4 < .1$.

At significant level $\alpha = .05$, we cannot reject $H_0 : \beta_2 = 0$, $H_0 : \beta_3 = 0$ or $H_0 : \beta_4 = 0$, but we should reject $H_0 : \beta_1 = 0$

OR: In STATA,
After inputting the data,

-. regress y x1 x2 x3 x4

| Source | SS | df | MS |
|---|---|---|---|
| Model | 207.457503 | 4 | 51.8643757 |
| Residual | 98.8125052 | 11 | 8.98295502 |
| Total | 306.270008 | 15 | 20.4180005 |

| | |
|---|---|
| Number of obs = | 16 |
| F( 4, 11) = | 5.77 |
| Prob > F = | 0.0094 |
| R-squared = | 0.6774 |
| Adj R-squared = | 0.5600 |
| Root MSE = | 2.9972 |

| y | Coef. | Std. Err. | t | P>\|t\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| x1 | -3.1375 | .7492895 | -4.19 | 0.002 | -4.786675 | -1.488325 |
| x2 | -1.2125 | .7492895 | -1.62 | 0.134 | -2.861675 | .4366749 |
| x3 | -.1625 | .7492895 | -0.22 | 0.832 | -1.811675 | 1.486675 |
| x4 | -1.275 | .7492895 | -1.70 | 0.117 | -2.924175 | .3741751 |

```
_cons |    21.125    .7492895    28.19    0.000     19.47583    22.77417
------------------------------------------------------------------------------

. test x1
 ( 1)  x1 = 0

       F(  1,    11) =   17.53
            Prob > F =    0.0015

. test x2
 ( 1)  x2 = 0

       F(  1,    11) =    2.62
            Prob > F =    0.1339

. test x3
 ( 1)  x3 = 0

       F(  1,    11) =    0.05
            Prob > F =    0.8323

. test x4
 ( 1)  x4 = 0

       F(  1,    11) =    2.90
            Prob > F =    0.1169
```

All the results correspond to what we got before.

Notice the relationship between F-statistic and t-statistic: $F = t^2$.

**11.80**

The F statistics is $F = \frac{\frac{SSE_1 - SSE_2}{2}}{\frac{SSE_2}{200-5}} = \frac{\frac{795.23-783.9}{2}}{\frac{783.9}{200-5}} = 1.41$.

The critical value is $F_{0.05}(2,195) = 3.00 > 1.41$, so we do not reject $H_0$: salary is not dependent on sex.

**11.82**

(a) Use the coding $X_1^* = \frac{X_1 - 65}{15}, X_2^* = \frac{X_2 - 200}{100}$.

Then $X_1 = 50, 80$ correspond to $X_1^* = -1, 1$, while $X_2 = 100, 200, 300$ correspond to $X_2^* = -1, 0, 1$.

Now,

$$Y=\begin{pmatrix}21\\23\\26\\22\\23\\28\end{pmatrix}, X^*=\begin{pmatrix}1&-1&-1&1\\1&-1&0&0\\1&-1&1&1\\1&1&-1&0\\1&1&0&1\\1&1&1&0\end{pmatrix}\Rightarrow X^{*\prime}X^*=\begin{pmatrix}6&&&4\\&6&&\\&&4&\\6&&&4\end{pmatrix}, \left(X^{*\prime}X^*\right)^{-1}=\begin{pmatrix}\frac{1}{2}&&&-\frac{1}{2}\\&\frac{1}{6}&&\\&&\frac{1}{4}&\\-\frac{1}{2}&&&\frac{3}{4}\end{pmatrix},$$

$$X^{*\prime}Y=\begin{pmatrix}143\\3\\11\\97\end{pmatrix}, \hat{\beta}^*=\begin{pmatrix}23\\.5\\2.75\\1.25\end{pmatrix}$$

Finally, the least squares equation is:

$$\hat{y}=23+.5\left(\frac{x_1-65}{15}\right)+2.75\left(\frac{x_2-200}{100}\right)+1.25\left(\frac{x_2-200}{100}\right)^2=20.33+.0333x_1-.0225x_2+.000125x_2^2$$

(b)

The hypothesis of interest is: $H_0:\beta_3=0$, is equivalent to a test of $H_0:\beta_3^*=0$ since $\beta_3=\left(\frac{1}{100}\right)^2\beta_3^*$.

Using the coded calculations, we have $SSE=Y'Y-\hat{\beta}^{*\prime}X'Y=1$ and $s^2=\frac{SSE}{n-4}=.5$. The test statistic is

$$t=\frac{\hat{\beta}_3^*}{\sqrt{c_{33}s^2}}=\frac{1.25}{\sqrt{\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)}}=2.042.$$

The RR with $\alpha=.05$ is $|t|>4.303$, and $H_0$ is not rejected. The quandratic temperature effect is not significant.

(c)

In order to test $H_0:\beta_2=\beta_3=0$, or equivalently, $H_0:\beta_2^*=\beta_3^*=0$, both complete and reduced models are fitted. For the complete model, $SSE_2=1$ with d.f.=2. (from part b) For the reduced model, columns 3 and 4 of $X^*$ matrix are omitted and $\left(X^{*\prime}X^*\right)^{-1}=\begin{pmatrix}\frac{1}{6}&\\&\frac{1}{6}\end{pmatrix}, X^{*\prime}Y=\begin{pmatrix}143\\3\end{pmatrix}\Rightarrow SSE_1=33.33$ with d.f.=4

The test statistic is then $F=\frac{\frac{SSE_1-SSE_2}{2}}{\frac{SSE_2}{2}}=\frac{\frac{33.33-1}{2}}{\frac{1}{2}}=32.23$. The RR with $\alpha=.05$ is $F>F_{2,2}=19.00$, and the null hypothesis is rejected. Temperature does affect yield.