11.10

a. $\sum x_i = 3.25, \quad \sum y_i = 7.55, \quad \sum x_i y_i = 2.1825,$

$\sum x_i^2 = 1.2625, \quad \sum y_i^2 = 6.0725, \quad n = 10$

So, $S_{xy} = -.27125, \quad S_{xx} = .20625$.

Then $\hat{\beta}_1 = \dfrac{S_{xy}}{S_{xx}} = -1.31515, \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = 1.182424$.

OR: Use STATA

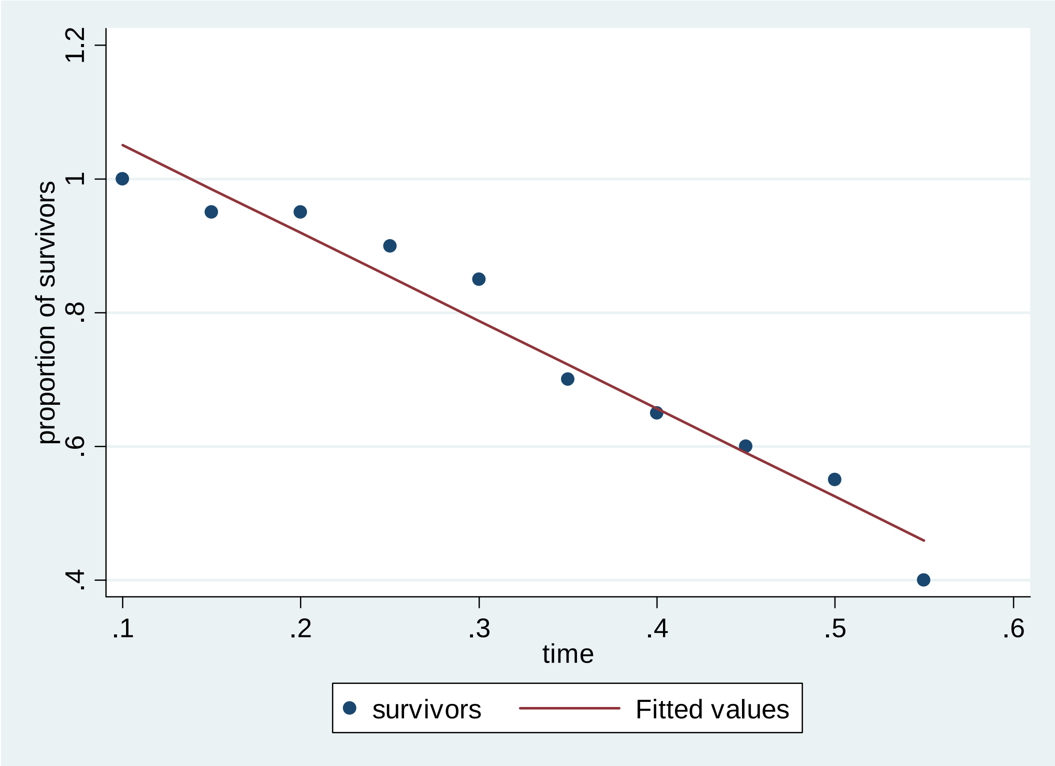
Put the data into Excel and save as .csv file. Import the data by clicking File->Import->ASCII data created by a spread sheet, or using command insheet directly.

```
. insheet using "C:\Documents and Settings\Sara\Desktop\sunfish.csv"
(2 vars, 10 obs)

. regress survivors time

      Source |       SS       df       MS              Number of obs =      10
-------------+------------------------------           F(  1,     8) =  183.94
       Model |  .356734825     1  .356734825           Prob > F      =  0.0000
    Residual |  .015515153     8  .001939394           R-squared     =  0.9583
-------------+------------------------------           Adj R-squared =  0.9531
       Total |  .372249978     9  .041361109           Root MSE      =  .04404

------------------------------------------------------------------------------
   survivors |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        time |  -1.315151   .0969697   -13.56   0.000    -1.538764   -1.091539
       _cons |   1.182424    .034455    34.32   0.000     1.102971    1.261878
------------------------------------------------------------------------------
```

So, $\hat{\beta}_1 = -1.31515, \hat{\beta}_0 = 1.182424$.

b.

```
. predict yhat
(option xb assumed; fitted values)

. scatter survivors time || line yhat time, xtitle(time) ytitle(proportion of survivors)
```



The regression line fits the data very well.

11.36

Refer to Exercise 11.10, calculate $S_{yy} = .37225$, $\bar{x} = .325$.

Then, $SSE = .0155$, and $S^2 = .0019394$.

Also, you can find these values in STATA.

From the result of the simple regression in 11.10, we see that $S^2 = .0019394$.

. summarize time

Variable	Obs	Mean	Std. Dev.	Min	Max
time	10	.325	.1513825	.1	.55

So, $S_{xx} = 0.1513825 \wedge 2 * (n-1) = 0.1513825 \wedge 2 * 9 = 0.20625$.

For $x = .3, \hat{y} = 1.182424 - 1.31515 * (.3) = .7878$

Use the formula on P560, we have the 90% confidence interval for $E(Y)$ is

$$.7878 \pm 1.86\sqrt{.0019394}\sqrt{\frac{1}{10}+\frac{(.3-.325)^2}{.20625}} = .7878 \pm .0263.$$

11.43

Refer to Exercise 11.10. When $x=.6, \hat{y}=1.182424-1.31515*(.6)=.3933$.

Use the formula on P564, we have the 95% confidence interval for Y at $x=.6$ is

$$.3933 \pm 2.306\sqrt{.0019394}\sqrt{1+\frac{1}{10}+\frac{(.6-.325)^2}{.20625}} = .3933 \pm .12$$, equivalently, $[.27,.51]$.

11.56

Fit the linear model: $y=\beta_0+\beta_1 x+\beta_2 x^2$ using the method of least squares.

For this exercise:

$$X = \begin{bmatrix} 1 & -3 & 9 \\ 1 & -2 & 4 \\ 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix}, Y = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \\ -1 \\ 0 \\ 0 \end{bmatrix}.$$

In MATLAB, calculate

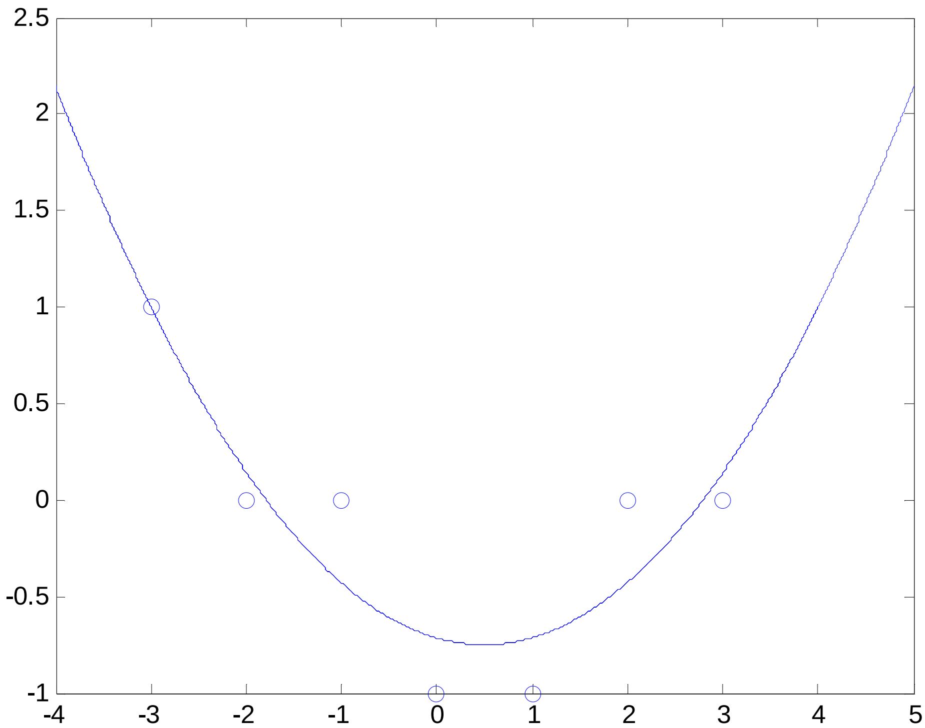
$$X'Y = \begin{bmatrix} -1 \\ -4 \\ 8 \end{bmatrix}, X'X = \begin{bmatrix} 7 & 0 & 28 \\ 0 & 28 & 0 \\ 28 & 0 & 196 \end{bmatrix}, (X'X)^{-1} = \begin{bmatrix} .3333 & 0 & -.0476 \\ 0 & .0357 & 0 \\ -.0476 & 0 & .0119 \end{bmatrix}.$$

So, $\hat{\beta} = (X'X)^{-1}X'Y = \begin{bmatrix} -.7143 \\ -.1429 \\ .1429 \end{bmatrix}$.

The fitted parabola is $\hat{y} = -.71 - .14x + .14x^2$.

$$SSE = Y'Y - \hat{\beta}'X'Y = .5714 \Rightarrow \hat{\sigma}^2 = SSE/(n-3) = .14286.$$

The fitted parabola and raw data are shown as the following:


2.5
2
1.5
1
0.5
0
-0.5
-1
-4
-3
-2
-1
0
1
2
3
4
5

11.51 a. Let $W = \ln Y$ so that $E(W) = \ln\alpha_0 - \alpha_1 x = \beta_0 + \beta_1 x$, where $\ln\alpha_0 = \beta_0$ & $-\alpha_1 = \beta_1$.

After calculating individual values of W for each of the $n=10$ observations, we have

$\Sigma x = 55$, $\Sigma w = 35.505412$, $\Sigma xw = 194.49729$,

$\Sigma x^2 = 385$, $\Sigma w^2 = 126.07188$, $n = 10$,

$S_{xw} = -.782481$, $S_{xx} = 82.5$, $S_{ww} = .008448$

Hence, $\hat{\beta}_1 = \frac{S_{xw}}{S_{xx}} = -\frac{.782481}{82.5} = -.00948$

& $\hat{\beta}_0 = \bar{w} - \hat{\beta}_1\bar{x} = 3.6027$

Transforming to the original variables, we have

$\hat{\alpha}_1 = -\hat{\beta}_1 = .00948$

& $\hat{\alpha}_0 = e^{\hat{\beta}_0} = 36.70$

and the prediction equation is $\hat{y} = 36.70e^{-.00948x}$

b. In order to find a CI for α_0, we first find a CI for β_0 & then transform the endpoints of the interval.

The LSE of β_0 is $\hat{\beta}_0$, which has variance

$V(\hat{\beta}_0) = \frac{\sigma^2\Sigma x_i^2}{n\Sigma(x_i-\bar{x})^2} = \frac{\sigma^2\Sigma x_i^2}{nS_{xx}}$ (given in Section 11.5)

It's necessary to calculate

$SSE = S_{ww} - \hat{\beta}_1 S_{xw} = .008448 - (-.00948)(-.782481)$

$= .0010265$

& $s^2 = SSE/_{n-2} = .0001283$

Then 90% CI for β_0 is

$\hat{\beta}_0 \pm t_{.05,8}\sqrt{V(\hat{\beta}_0)} = 3.6027 \pm 1.86\sqrt{.0001283\left[\frac{385}{10(82.5)}\right]}$

yielding the interval [3.5883, 3.6171]
Transforming, we have the interval for α_0 as
$[e^{3.5883}, e^{3.6171}] = [36.17, 37.23]$

11.52 Similar to Exercise 11.51
Notice $\ln(e^{-\alpha_0 X^{\alpha_1}}) = -\alpha_0 X^{\alpha_1}$,
& $\ln[-\ln(e^{-\alpha_0 X^{\alpha_1}})] = \ln\alpha_0 + \alpha_1 \ln X$
Therefore, we would expect $\ln(-\ln y)$ to be linear in $\ln x$
Define $V = \ln(X)$, $Z = \ln(-\ln Y)$, $\beta_0 = \ln\alpha_0$, $\beta_1 = \alpha_1$.
We fit the model $E(Z) = \beta_0 + \beta_1 V$
(We are assuming the error remains additive when we take two natural logs, this may not be a valid assumption)
To fit the model, we have
$\sum v = -10.15243$, $\sum z = -13.1544$, $\sum vz = 18.521577$
$\sum v^2 = 12.977909$, $\sum z^2 = 28.50516$, $n = 9$,
$S_{vz} = 3.6828016$, $S_{vv} = 1.51548$
$\Rightarrow \quad \hat{\beta}_1 = S_{vz}/S_{vv} = 2.4142$

$\hat{\beta}_0 = \bar{z} - \hat{\beta}_1 \bar{v} = 1.2617$
Transforming, we have $\hat{\alpha}_1 = \hat{\beta}_1 = 2.4142$
$\hat{\alpha}_0 = e^{\hat{\beta}_0} = 3.5315$
The predictor is $\hat{y} = \exp(-3.5315\, x^{2.4142})$