Likelihood ratio test

The general form of likelihood ratio test

$$H_0 : \theta \in \Theta_0, \quad H_a : \theta \in \Theta_a$$

- The test statistics

$$\lambda \doteq \frac{\max_{\theta \in \Theta_0} L(x_1, x_2, \ldots, x_n; \theta)}{\max_{\theta \in \Theta} L(x_1, x_2, \ldots, x_n; \theta)}$$

  where $\Theta = \Theta_0 \cup \Theta_a$.

- Rejection region $\{\lambda \leq k\}$ for some k.

REMARK: Θ_0 and Θ_a may contain nuisance parameters. And $0 \leq \lambda \leq 1$.

Example

1. Suppose $Y_1, Y_2, \ldots, Y_n$ are iid samples from Bernoulli with parameter p.

$$H_0 : p = p_0, \qquad H_a : p > p_0.$$

2. Suppose $Y_1, Y_2, \ldots, Y_n$ are iid samples from $N(\mu, \sigma^2)$. μ and σ^2 are both unknown. We want to test

$$H_0 : \mu = \mu_0, \qquad H_a : \mu > \mu_0.$$

Find the appropriate likelihood ratio test.

Large sample distribution of λ

Theorem: When n is large, the distribution $-2\ln(\lambda)$ under H_0 is approximately χ^2 with degree of freedom equal

$$\text{number of free parameters in } \Theta - \text{number of free parameters in } \Theta_0.$$

The Rejection region with significance level α is just

$$\{-2\ln(\lambda) \geq \chi^2_\alpha\}.$$