HW5.

Using Table 4,

4.46. a) The area between $z=0$ and $z=1.2$ is $(0.5-0.1151) = 0.3849$.

b) The area between $z=0$ and $z=-0.9$ is $(0.5-0.1841) = 0.3159$.

c) The area between $z=0.3$ and $z=1.56$ is $(0.3821-0.0594) = 0.3227$.

d) The area between $z=-0.2$ and $z=0.2$ is $(1-2\times 0.4207) = 0.1586$.

e) The area between $z=-1.56$ and $z=-0.2$ is $(0.4207-0.0594) = 0.3613$.

4.47. a) $z_0=0$

b) $z_0=1.1$

c) $z_0=1.645$

d) $z_0=2.575$

Remark: If the exact probability cannot be found in the table, we may choose to search for the probability closest to the one desired and perform an interpolation (usually linear interpolation is okay!) that will determine the exact value of z_0.

4.50 $\mu=400 \quad \sigma=20$.

$z_1 = \frac{450-400}{20} = 2.5$. Then

$P(Y>450) = P(Z>2.5) \underset{\uparrow}{=} 0.0062$.

Table 4

note: Y denotes "exam score"

4.60 a) $P(Y>72) = P(Z>\frac{72-78}{6}) = P(Z>-1) = 1-P(Z>1)$
$= 0.8413$;

b) We seek c s.t. $P(Y>c)=0.1 \Leftrightarrow 0.1 = P(Y>c) = P(Z>\frac{c-78}{6}) = P(Z>z_0)$

$\Rightarrow z_0 = 1.28 = \frac{c-78}{6} \Rightarrow c = 85.68$;

c) We seek c s.t. $P(Y>c) = 0.281 \Rightarrow$
$0.281 = P(Y>c) = P(Z>\frac{c-78}{6}) = P(Z>z_0)$
$\Rightarrow z_0 = 0.58 = \frac{c-78}{6} \Rightarrow c = 81.48$;

d) $P(Z<\frac{c-78}{6}) = 0.25$

Now. $P(Z>0.67) = 0.25 \Rightarrow P(Z<-0.67) = 0.25$
$\Rightarrow \frac{c-78}{6} = -0.67 \Rightarrow c = 73.98$,

We must now find $P(Y>73.98+5) = P(Y>78.98)$
$= P(Z>\frac{78.98-78}{6}) = P(Z>0.16)$
$= 0.4364$;

e) $P(Y>84 \mid Y>72) = \frac{P(Y>84)}{P(Y>72)} = \frac{P(Z>\frac{84-78}{6})}{P(Z>\frac{72-78}{6})} = \frac{P(Z>1)}{P(Z>-1)}$

$= \frac{P(Z>1)}{1-P(Z>1)} = \frac{0.1587}{0.8413} = 0.1886$.

4.77 $f(y) = \frac{1}{4} e^{-\frac{y}{4}}$ for $y \geq 0$ Then $P(Y>4) = \int_4^\infty \frac{1}{4} e^{-\frac{y}{4}} dy$

$= -e^{-\frac{y}{4}} \Big|_4^\infty = e^{-1} \doteq 0.368$。

4.142 a) $f(y) = 2e^{-2y}$ $y > 0$

$$P(Y > \tfrac{1}{4}) = \int_{\frac{1}{4}}^\infty 2e^{-2y} dy = -e^{-2y} \Big|_{\frac{1}{4}}^\infty = e^{-\frac{1}{2}} \doteq 0.61;$$

b) $P(Y > \frac{1}{3} + \frac{1}{4} \mid Y > \frac{1}{4}) = P(Y > \frac{1}{3}) = \int_{\frac{1}{3}}^\infty 2e^{-2y} dy$

see Text book P.179.

$= -e^{-2y} \Big|_{\frac{1}{3}}^\infty = e^{-\frac{2}{3}} \doteq 0.51$

c) $\mu = \int_0^\infty z \, f(Y = z + \frac{1}{4} \mid Y > \frac{1}{4}) \, dz$

z denotes "waiting time"

$f(Y = z + \frac{1}{4} \mid Y > \frac{1}{4}) = 2e^{-2z}$

so $\mu = \int_0^\infty z \cdot 2e^{-2z} dz = \frac{1}{2}$ 。

5.2 a) The sample space for the toss of three balanced coins, the values for Y_1 and Y_2 at each outcome and the probability of each outcome are given below:

outcomes	(Y_1, Y_2)	probability
HHH	(3, 1)	$\frac{1}{8}$
HHT	(2, 1)	$\frac{1}{8}$
HTH	(2, 1)	$\frac{1}{8}$
HTT	(1, 1)	$\frac{1}{8}$
THH	(2, 2)	$\frac{1}{8}$
THT	(1, 2)	$\frac{1}{8}$
TTH	(1, 3)	$\frac{1}{8}$
TTT	(0, -1)	$\frac{1}{8}$

Y_2 \ Y_1	0	1	2	3
-1	$\frac{1}{8}$	0	0	0
1	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
2	0	$\frac{1}{8}$	$\frac{1}{8}$	0
3	0	$\frac{1}{8}$	0	0

b). $F(2,1) = P(Y_1 \le 2, Y_2 \le 1) = p(0,-1) + p(1,1) + p(2,1)$

$= \frac{1}{8} + \frac{1}{8} + \frac{1}{4} = \frac{1}{2}$ 。

5.6 a). We must have $F(\infty,\infty) = \int_0^1 \int_0^1 k y_1 y_2 \, dy_1 \, dy_2 = 1$

Then

$\int_0^1 \int_0^1 k y_1 y_2 \, dy_1 \, dy_2 = k \int_0^1 y_2 \cdot \left(\frac{y_1^2}{2}\right)\Big|_0^1 dy_2 = \frac{k}{4} = 1$

$\Rightarrow k = 4$;

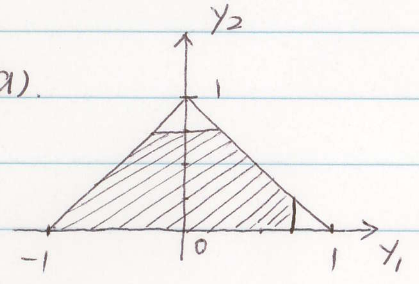
b). $F(y_1, y_2) = \int_0^{y_2} \int_0^{y_1} 4 t_1 t_2 \, dt_1 \, dt_2 = y_1^2 y_2^2$

for $0 \le y_1 \le 1$ and $0 \le y_2 \le 1$ 。 Recall that

$$F(y_1, y_2) = \begin{cases} 0 & \text{for } y_1 \le 0 \text{ or } y_2 \le 0 \\ 1 & \text{for } y_1 \ge 1 \text{ and } y_2 \ge 1 \\ y_2^2 & \text{for } y_1 \ge 1 \text{ and } 0 \le y_2 \le 1 \\ y_1^2 & \text{for } 0 \le y_1 \le 1 \text{ and } y_2 \ge 1 \end{cases}$$;

c). $P(Y_1 \le \frac{1}{2}, Y_2 \le \frac{3}{4}) = F(\frac{1}{2}, \frac{3}{4}) = (\frac{1}{2})^2 (\frac{3}{4})^2 = \frac{9}{64}$ 。

5.9 a).

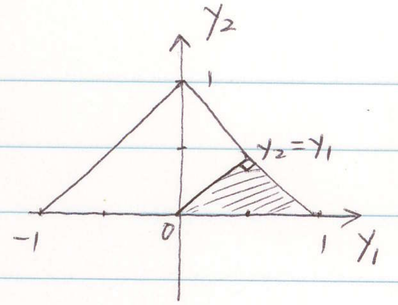


$P(Y_1 \le \frac{3}{4}, Y_2 \le \frac{3}{4}) =$ the shaded area

$= 1 - \frac{1}{4} \times \frac{1}{4} \times \frac{1}{2} - \frac{1}{4} \times \frac{1}{4} \times \frac{1}{2} \times 2$

$= 1 - \frac{1}{32} - \frac{2}{32} = \frac{29}{32}$;

5.9 b)



$P(Y_1 - Y_2 \geq 0) =$ the shaded area

$$= \int_0^{\frac{1}{2}} \int_{y_2}^{1-y_2} dy_1\, dy_2 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

5.12 a) clearly $f(y_1, y_2) \geq 0$ Then $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(y_1, y_2)\, dy_1\, dy_2$

$$= \int_0^1 \int_{y_1}^{2-y_1} 6 y_1^2 y_2\, dy_2\, dy_1$$

$$= \int_0^1 3 y_1^2 [(2-y_1)^2 - y_1^2]\, dy_1$$

$$= \int_0^1 (12 y_1^2 - 12 y_1^3)\, dy_1 = 4 - 3 = 1;$$

note: by $\begin{cases} 0 \leq y_1 \leq y_2 \\ y_1 + y_2 \leq 2 \end{cases}$

$\Leftrightarrow y_1 \leq y_2 \leq 2 - y_1$

and $\Rightarrow y_1 \leq 2 - y_1 \Rightarrow y_1 \leq 1$

b) $P(Y_1 + Y_2 < 1) = \int_0^{\frac{1}{2}} \int_{y_1}^{1-y_1} 6 y_1^2 y_2\, dy_2\, dy_1$

$$= \int_0^{\frac{1}{2}} 3 y_1^2 [(1-y_1)^2 - y_1^2]\, dy_1$$

$$= \int_0^{\frac{1}{2}} (3 y_1^2 - 6 y_1^3)\, dy_1 = \frac{1}{8} - \frac{6}{64} = \frac{1}{16} \times \frac{1}{2} = \frac{1}{32}$$