10.23 Two binomial populations are involved.

To test $H_0: p_1 = p_2$ vs. $H_a: p_1 > p_2$ (one-tail)

The test statistic $Z = \dfrac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}}$

To evaluate the denominator, we must estimate p_1, p_2.

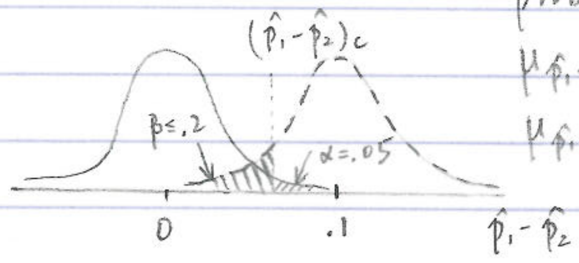
Under H_0: $p_1 = p_2$, the best estimate for this common value is $\hat{p} = \dfrac{y_1 + y_2}{n_1 + n_2} = \dfrac{46 + 34}{200 + 200} = 0.2$

$$\Rightarrow Z = \frac{\hat{p}_1 - \hat{p}_2 - 0}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n} + \frac{1}{n}\right)}} = \frac{\frac{46}{200} - \frac{34}{200}}{\sqrt{(0.2)(0.8)\left(\frac{1}{100}\right)}} = 1.5$$

Rejection region: under $\alpha = .05$, H_0 will be rejected if $Z > 1.645$ (one-tail)

So, we fail to reject H_0. There is insufficient evidence to support the researcher's belief

10.30



The left figure represents the two probability distribution, one assuming $\mu_{\hat{p}_1 - \hat{p}_2} = p_1 - p_2 = 0$ & one assuming $\mu_{\hat{p}_1 - \hat{p}_2} = p_1 - p_2 = .1$.

The right curve is the true distribution of $\hat{p}_1 - \hat{p}_2$ & thus any probabilities we wish to calculate concerning the random variable should be calculated as areas under the curve to the right.

We need to find a common sample sise such that $\alpha = P(\text{reject } H_0 \mid H_0 \text{ true}) = .05$ & $\beta = P(\text{accept } H_0 \mid H_0 \text{ false}) \le .2$

For $\alpha = .05$, the critical value $(p_1 - p_2)_c$ is 1.645 (recall Ex 10.23), then $1.645 = (\hat{p}_1 - \hat{p}_2)_c - 0 \Big/ \sqrt{\frac{p_1 q_1}{n} + \frac{p_2 q_2}{n}}$

For β = the area under the curve to the right from $-\infty$ to $(p_1-p_2)c$, since $\beta = .2$, we have z value $= -.84$.

$$\Rightarrow \quad -.84 = \frac{(\hat{p}_1-\hat{p}_2)c - .1}{\sqrt{\frac{p_1q_1}{n_1}+\frac{p_2q_2}{n_2}}} \qquad (2)$$

Combine (1) & (2), $\quad 2.485 = \dfrac{1}{\sqrt{\frac{p_1q_1}{n_1}+\frac{p_2q_2}{n_2}}}$

Note: (1) $n_1 = n_2 = n$

(2) max value of $pq = p(1-p)$ is $.25$ where $p = .5$. Since p_1 & p_2 are unknown, the use of $p = .5$ will provide a valid (although may be larger than necessary) sample size

Then, $\quad 2.485 = \dfrac{1}{\sqrt{(.5)(.5)\left(\frac{1}{n}+\frac{1}{n}\right)}} \quad \Rightarrow \sqrt{n} = 17.57 \quad \Rightarrow n = 308.76$

So, the common sample size for the researcher's test should be 309.

10.33 (a) Let μ_1 be the average manual dexterity score for those that participated in sports & μ_2 for those that did not.

Then $H_0: \mu_1 = \mu_2$ VS $H_a: \mu_1 > \mu_2$. Test statistic is

$$z = \frac{(\bar{y}_1-\bar{y}_2)-0}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}} = \frac{32.19-31.68}{\sqrt{\frac{(4.34)^2+(4.56)^2}{37}}} = .49$$

The RR with $\alpha = .05$ is $z > 1.645$. So, H_0 is not rejected. There is insufficient evidence to indicate $\mu_1 > \mu_2$

(b) The RR, written in terms of $\bar{Y}_1 - \bar{Y}_2$, is

$$Z = \frac{\bar{Y}_1 - \bar{Y}_2}{\hat{\sigma}_{\bar{Y}_1 - \bar{Y}_2}} > 1.645$$

or $$\bar{Y}_1 - \bar{Y}_2 > 1.645 \sqrt{\frac{(4.34)^2 + (4.56)^2}{37}} = 1.702$$

Then $\beta = P(\text{accept } H_0 \mid \mu_1 - \mu_2 = 3) = P(\bar{Y}_1 - \bar{Y}_2 < 1.702 \mid \mu_1 - \mu_2 = 3)$

$$= P\left(Z < \frac{1.702 - 3}{\hat{\sigma}_{\bar{Y}_1 - \bar{Y}_2}}\right) = P(Z < -1.25) = .1056$$

10.34 Using the procedure discussed following Example 10.8, we can write $\alpha = P(\bar{Y}_1 - \bar{Y}_2 > k \text{ when } \mu_1 - \mu_2 = 0) = P\left(Z > \frac{k - 0}{\sqrt{\frac{\sigma_1^2 + \sigma_2^2}{n}}}\right)$

$$\Rightarrow Z_\alpha = \frac{k\sqrt{n}}{\sqrt{\sigma_1^2 + \sigma_2^2}} \qquad ①$$

Also $\beta = P(\bar{Y}_1 - \bar{Y}_2 \le k \mid \mu_1 - \mu_2 = 3) = P\left(Z \le \frac{k - 3}{\sqrt{\frac{\sigma_1^2 + \sigma_2^2}{n}}}\right)$

$$\Rightarrow -Z_\beta = \frac{(k-3)\sqrt{n}}{\sqrt{\sigma_1^2 + \sigma_2^2}} \qquad ②$$

Combine ① & ② to eliminate k, we get

$$Z_\alpha \sqrt{\frac{\sigma_1^2 + \sigma_2^2}{n}} = 3 - Z_\beta \sqrt{\frac{\sigma_1^2 + \sigma_2^2}{n}}$$

Solving for n, we have $n = \dfrac{[2(1.645)]^2 \left[(4.34)^2 + (4.56)^2\right]}{3^2} = 47.66$

or $n = 48$ to provide the given levels of α and β.

10.42 (a) Let p_1 & p_2 be the proportions (attending vs. not attending) who were using safety seats 4 to 6 weeks after birth.

From the study, $n_1 = 78$, $\hat{p}_1 = .96$; $n_2 = 136 - 78 = 58$, $\hat{p}_2 = .78$

Then $\hat{p} = \dfrac{78(.96) + 58(.78)}{136} = .883$

The hypothesis to be tested is $H_0: p_1 = p_2$ VS $H_a: p_1 > p_2$

The test statistic $z = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \dfrac{.96 - .78}{\sqrt{(.883)(.117)\left(\frac{1}{78} + \frac{1}{58}\right)}}$

$= 3.23 > 1.645$

So, reject H_0. There is evidence that the lecture is effective

(b) $p\text{-value} = P(z > 3.23) < 0.00135$