AM165 HW10

10.2) The test statistic Y has a binomial distribution with $n=20$ and p.

a. A Type I error occurs if the experimenter concluded that the drug dosage level induces sleep in less than 80% of the people suffering from insomnia when, in fact, drug dosage level does induce sleep in 80% of insomniacs.

b. $\alpha = P(\text{reject } H_0 \mid H_0 \text{ true}) = P(Y \leq 12 \mid p=0.8)$

$= 0.032$ using Table 1, Appendix III.

c. A type II error would occur if the experimenter concluded that the drug dosage level induces sleep in 80% of the people suffering from insomnia when, in fact, fewer than 80% experience relief.

d. If $p=0.6$,

$$\beta = P(\text{accept } H_0 \mid H_0 \text{ false}) = P(Y>12 \mid p=0.6)$$

$$= 1 - P(Y \leq 12 \; p=0.6) = 1 - 0.584 = 0.416$$

e. If $p=0.4$ then

$$\beta = P(Y>12 \mid p=0.4) = 1 - P(Y \leq 12 \mid p=0.4)$$

$$= 1 - 0.979 = 0.021$$

10.4) a. A Type I error occurs if we conclude that the proportion of ledger sheets with errors is larger than 0.05 when, in fact, the proportion is 0.05.

b. By the scheme being used, we will reject for the following situations:

(NOTE: NE = no error, E = error)

sheet 1	sheet 2	sheet 3
NE	NE	.
NE	E	NE
E	NE	NE
E	E	NE

Thus $\alpha = (0.95)^2 + 2(0.05)(0.95)^2 + (0.05)^2(0.95)$
$= 0.995125$.

c. A Type II error occurs if we conclude that the proportion of ledger sheets with errors is 0.05 when in fact the proportion is larger than 0.05.

d. $\beta = P(\text{accept } H_0 \text{ when } H_a \text{ is true}) = P(\text{accepting } H_0 \mid p = p_a) = 2p_a^2(1-p_a) + p_a^3$. Since we reject if we observe E, E, E or NE, E, E or E, NE, E.

10.7) a. Since it is necessary to test a claim that the average amount saved, μ is \$900, the hypothesis to be tested is two-tailed:

$$H_0: \mu = 900 \quad \text{vs.} \quad H_a: \mu \neq 900.$$

b. The rejection region with $\alpha = 0.01$ is determined by a critical value of z such that $P[|z| > z_0] = 0.01$. This value is $z_0 = 2.58$ and the rejection region is $|z| > 2.58$.

c. The test statistic is

$$z = \frac{\bar{y} - \mu}{\frac{\sigma}{\sqrt{n}}} \doteq \frac{\bar{y} - \mu}{\frac{s}{\sqrt{n}}} = \frac{885 - 900}{\frac{50}{\sqrt{35}}} = -1.77$$

d. The observed value $z = -1.77$ does not fall in the rejection region, and H_0 is not rejected. We cannot conclude that the average savings is different than claimed.

10.11) $H_0: \mu_1 - \mu_2 = 0$ $\quad H_a: \mu_2 - \mu_1 \neq 0$ The test statistic and rejection region are

$$z = \frac{1.65 - 1.43}{\sqrt{\frac{(0.26)^2}{30} + \frac{(0.22)^2}{35}}} = 3.65$$

RR: Reject H_0 if $|z| > 2.575$

Conclusion: Reject H_0 at $\alpha = 0.01$. The soils do appear to differ with respect to average shear strength, at the 1% significance level.

10.14) a. define p as the proportion of college students aged 30 years or more, then we test

$H_0: p = 0.25$ vs. $H_a: p \neq 0.25$

The test statistic is

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{\frac{98}{300} - 0.25}{\sqrt{\frac{(0.25)(0.75)}{300}}} = 3.07$$

and the rejection region, with $\alpha = 0.05$ is $|z| > 1.96$. H_0 is rejected and we conclude that the 25% figure is not accurate.

b. Yes. the results do give evidence that the columnist's claim is too low.