

AM 121: Homework # 6 (Do not hand in)

The book we refer to is Hillier & Lieberman, *Introduction to Operations Research* (8th Edition).

Do not hand in this homework. Please do it by yourself first and then double check with the solution that has already been posted on web.

1. Problem 10-3.7. This is a resource allocation problem with multiplicative (instead of additive) benefit. Use dynamic programming to write down the DPE. Find the optimal solution.
2. Problem 11.1-2 (a).
3. Problem 11.3-1. Only do the formulation part.
4. Problem 11.3-4 (a).

The following is a difficult optimization problem. Try to solve it using the dynamic programming approach hinted in the statement.

- During any year, I can consume any amount that does not exceed my current wealth. If I consume c dollars a year, I earn c^a units of happiness (utility). Assume $0 < a < 1$. By the beginning of the next year, the previous year's ending wealth grows by a factor k .
 1. Suppose I currently have w_0 dollars, and want to maximize the total utility earned during the next N years. Formulate the problem as a dynamic programming problem. Clearly indicate the stage, the state, the evolution of system, the cost criteria.
 2. Let $f_n(w)$ be the maximum utility earned during years $n, n + 1, \dots, N$, given that I have w dollars at the beginning of year n ; and let $c_n^*(w)$ be the optimal amount that should be consumed during year n to attain $f_n(w)$. By working backwards, show that for appropriately chosen constants a_n and b_n ,

$$f_n(w) = b_n w^a, \quad c_n^*(w) = a_n w.$$

Interpret these results.