Solns to 5^{th} Homework

14.3-1 Polar game

(a) pure strategies

Each strategy should be a consideration of both cases:Head/Tail, thus for player 1, choosing pass with head is not a strategy. But choosing pass with head and bet with tail is a strategy as told in hint. With this understanding, we have four pure strategies for player 1

1	pass with head	pass with tail
2	pass with head	bet with tail
3	bet with head	pass with tail
4	bet with head	bet with tail

For player 2 there are two pure strategies:

1	pass
2	call

(b) Payoff matrix

	1	2
1	-5	-5
2	0	-7.5
3	0	2.5
4	5	0

Each entry in the payoff matrix is the corresponding expected result of each pair of strategies.

(c) Unstable

Notice for player 1, strategy 1 is dominated by 3, and strategy 2 is dominated by 4. Thus the payoff matrix after elimination is:

	1	2
3	0	2.5
4	5	0

since $minmax = 2.5 \neq maxmin = 0$, there is no pure strategy saddle point. If player 1 start from any strategy, check out the serial corresponding strategy by player 1 and player 2 , we can see there is no equilibrium to be reached.

(d) Mixed strategy
If the (x_1, x_2) is the probability distribution of player 1, and (y_1, y_2) of player2, the expectation reward will be $\sum_{i=1}^{2}\sum_{j=1}^{2} x_i y_j b_{ij}$,
(1) $y_1 = 1, y_2 = 0, Payoff = 5x_2$
(2) $y_1 = 0, y_2 = 1, Payoff = 2.5x_2$
(3) $y_1 = .5, y_2 = .5, Payoff = 1.25x_1 + 2.5x_2$

2 Mo and Bo
We see the payoff matrix is :

25	-25
-1	1

since $minmax = 1 \neq maxmin = -1$, there is no pure strategy saddle point. Thus mixed strategy is needed. suppose strategy for Mo is (x_1, x_2), strategy for bo is (y_1, y_2), the expected win is $\sum_{i=1}^{2}\sum_{j=1}^{2} x_i y_j b_{ij} = y_1(25x_1 - x_2) + y_2(-25x_1 + x_2)$. For Mo if he want to maximize his win, he need to $maximize(min(25x_1 - x_2, -25x_1 + x_2))$, we know it is achieved when $x_1 = 1/26$, $x_2 = 25/26$.
For Bo the expected loss is $\sum_{i=1}^{2}\sum_{j=1}^{2} x_i y_j b_{ij} = x_1(25y_1 - 25y_2) + x_2(y_2 - y_1)$, if Bo want to minimize his loss, he need to $minimize(max(25y_1 - 25y_2, y_2 - y_1))$, which is achieved when $y_1 = y_2 = .5$.
Thus the optimal strategy for Mo is $(1/26, 25/26)$, and $(.5, .5)$ for Bo.

3 Football
The payoff matrix is:

1	8
10	0

a We see $minmax = 0$ and $maxmin = 1$, thus there is no saddle point.

b Using the same method in problem 2, supposing mixed strategy for offense is x_1, x_2, the offense side need to $maximize(min(x_1 + 10x_2, 8x_1))$, which is achieved when $x_1 = 10/17, x_2 = 7/17$.
For defense side, it's needed to do $minimize(max(y_1 + 8y_2, 10y_1))$, which is achieved when $y_1 = 8/17, y_2 = 9/17$. Thus the optimal strategy is $(10/17.7/17)$ for offense side, $(8/17, 9/17)$ for defense side.

c Pass against defense
The new payoff matrix is:

1	8
10+k	0

The same method is applied, the optimal strategy is $x_1 = (10+k)/(17+k), x_2 = 7/(10+k)$, we can see x_2 decrease when k increase.

4 Hide and seek
The payoff matrix is:

	b_{11}	b_{12}	b_{21}	b_{22}
row 1	1	2	0	0
row 2	0	0	3	4
column 1	1	0	3	0
column 2	0	2	0	4

The corresponding LP problem is:
$maxmize Z = x_5$, subject to

$$\begin{aligned} -x_1 - x_3 + x_5 &\leq 0 \\ -2x_1 - 2x_4 + x_5 &\leq 0 \\ -3x_2 - 3x_3 + x_5 &\leq 0 \\ -4x_2 - 4x_4 + x_5 &\leq 0 \\ x_1 + x_2 + x_3 + x_4 &\leq 1 \end{aligned}$$

with $x_1, x_2, x_3, x_4 \geq 0$ and no constraint on x_5.