Solns to 2nd Homework

1 (a) primal and dual

primal LP problem	dual LP problem
max $Z = x_1 + 3x_2$	min $W = -6y_1 + 2y_2$
$-3x_1 - x_2 \leq -6$	$y_1 \geq 0$
$2x_1 - x_2 = 2$	no sign constraint on y_2
$x_1 \geq 0$	$-3y_1 + 2y_2 \geq 1$
$x_2 \geq 0$	$-y_1 - y_2 \geq 3$

(b) primal and dual

primal LP problem	dual LP problem
max $Z = x_1 + 3x_2$	min $W = -6y_1 + 2y_2 - 2y_3$
$-3x_1 - x_2 \leq -6$	$y_1 \geq 0$
$2x_1 - x_2 \leq 2$	$y_2 \geq 0$
$-2x_1 + x_2 \leq -2$	$y_3 \geq 0$
$x_1 \geq 0$	$-3y_1 + 2y_2 - 2y_3 \geq 1$
$x_2 \geq 0$	$-y_1 - y_2 + y_3 \geq 3$

(c) Equivalence

For dual(a), as there is no constrain on y_2, we can let $y_2 = u - v$, with $u \geq 0, v \geq 0$, then we have:
objective min $w = -6y_1 + 2u - 2v$ with $y_1 \geq 0, u \geq 0, v \geq 0$ under the constraints:
$-3y_1 + 2u - 2v \geq 1$ and $-y_1 - 2u + 2v \geq 3$ which has the same form as in dual(b)

(d) Unbounded primal and infeasible dual

just plot the constraint picture of dual LP problem, we can check it out there is no feasible region at all. of course you can show it from the lemma.

2 Find optimal value use this table in book

Basic Variable	Row	Z	x	s	RHS
Z	(0)	1	$C_B^T B^{-1} A - c^T$	$C_B^T B^{-1}$	$C_B^T B^{-1} b$
x_B	:	0	$B^{-1} A$	B^{-1}	$B^{-1} b$

Now you see $C_B^T B^{-1} = (0.4, 1.4)$ and $b = (0.5, 0.5)$, thus optimal of the LP problem is $0.4 * 0.5 + 1.4 * 0.5 = 0.9$.

3 reconstruct LP problem

(a) find which is optimal
From above table, we know the first row now is $C_B^T B^{-1} A - c^T$, replace these vectors into it, you will see the first row is $(4.5, -2, 0, 0, 2.50)$ when you take the first B^{-1} into it, which says it's not optimal b/c the negative number.
And the result bringing the second B^{-1} into it is $(2, 8, 0, 5, 0, 0)$, which is the optimal case b/c it's not negative.

(b) recover the LP simplex table using above form. you can get

BV	row	Z	x_1	x_2	x_3	s_1	s_2	s_3	RHS
Z	0	1	2	8	0	5	0	0	150
x_3	1	0	1	2	1	1	0	0	130
s_2	2	0	1	-4	0	-2	1	0	0
s_3	3	0	1	4	0	0	0	1	20

See in the optimal case s_2 takes a value as 0, thus this LP problem is degenerate.

(c) Constraint
We see now $b = (R, 60, 20)$ and notice that $C_B^T B^{-1} A - c^T$, $B^{-1} A$ will not change when b changes. Thus in order to keep optimal, we only need to keep $B^{-1} b \geq 0$, to keep the solution feasible. Replace these vectors into this constraint, we have:
$-2R + 60 \geq 0$, so when $0 \leq R \leq 30$, the solution is still optimal.