AM 121: Homework # 3 (Due Oct 25, Tuesday)

The book we refer to is Hillier & Lieberman, *Introduction to Operations Research* (8th Edition).

1. Consider the following LP:

$$\text{Maximize} \qquad Z = x_1 + 3x_2$$

subject to constraints

$$\begin{aligned} 3x_1 + x_2 &\geq 6 \\ 2x_1 - x_2 &= 2 \end{aligned}$$

and $x_1, x_2 \geq 0$.

   (a) Find the dual of the above LP using the rules given in the lecture notes.

   (b) Now transform the primal LP to standard form (i.e., make all the functional constraints "$\leq$"). Write down the dual of this new LP using the rules given in the lecture notes.

   (c) Argue that the two dual LPs obtained in (a) and (b) are equivalent. (*Hint:* Any decison variable, say y, with no sign constraints can be written as $y = u - v$ with $u \geq 0$, $v \geq 0$).

   (d) Use graphical methods to argue that the primal is unbounded, and the dual is infeasible.

2. Consider the following LP,

$$\text{Maximize} \qquad Z = -x_1 + 5x_2$$

subject to constraints

$$\begin{aligned} x_1 + 2x_2 &\leq 0.5 \\ -x_1 + 3x_2 &\leq 0.5 \end{aligned}$$

and $x_1, x_2 \geq 0$. Suppose that Row (0) of the optimal tableau for this LP reads

Basic Variable	Row	Z	x_1	x_2	s_1	s_2	RHS
Z	(0)	1	0	0	0.4	1.4	??

Without going through the simplex algorithm, find the optimal value of the LP.

3. Consider the LP problem:

$$\text{Maximize} \qquad Z = 3x_1 + 2x_2 + 5x_3$$

subject to constraints

$$\begin{aligned} x_1 + 2x_2 + x_3 &\leq 30 \\ 3x_1 \qquad + 2x_3 &\leq 60 \\ x_1 + 4x_2 \qquad &\leq 20 \end{aligned}$$

and $x_1, x_2, x_3 \geq 0$. Someone tells you that the optimal solution has either

$$\text{basic variables } \mathbf{x_B} = [s_1, x_3, s_3] \text{ and } \mathbf{B}^{-1} = \begin{bmatrix} 1 & -0.5 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

or

$$\text{basic variables } \mathbf{x_B} = [x_3, s_2, s_3] \text{ and } \mathbf{B}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

but he cannot remember which.

(a) Without going through simplex algorithm, identify the correct set of basic variables for the optimal solution.

(b) Reconstruct the whole optimal table. From the optimal table, argue that the LP is degenerate.

(c) Suppose that the first constraint in the LP is now

$$x_1 + 2x_2 + x_3 \leq R.$$

For what range of R the basic variable $\mathbf{x_B}$ is still optimal (i.e. $\mathbf{x_B}$ is still the basic variables in the optimal solution)?

4. Problem 6.3.5.

5. Problem 6.7.3 (a), check whether the current set of optimal basic variables remains optimal. (b)(d)(e) check whether the current set of optimal basic variables remains optimal, and re-optimize if it's no longer optimal. Ignore parts (c)(f)(g).