

AM 121: Homework # 3 (Due Oct 10, Thursday)

The book we refer to is Hillier & Lieberman, *Introduction to Operations Research* (7th Edition).

1. Consider the following LP:

$$\text{Maximize } Z = x_1 + 3x_2$$

subject to constraints

$$3x_1 + x_2 \geq 6$$

$$2x_1 - x_2 = 2$$

and $x_1, x_2 \geq 0$.

- (a) Find the dual of the above LP using the rules given in the lecture notes.
- (b) Now transform the primal LP to standard form (i.e., make all the functional constraints “ \leq ”). Write down the dual of this new LP using the rules given in the lecture notes.
- (c) Argue that the two dual LPs obtained in (a) and (b) are equivalent. (*Hint:* Any decision variable, say y , with no sign constraints can be written as $y = u - v$ with $u \geq 0, v \geq 0$).
- (d) Use graphical methods to argue that the primal is unbounded, and the dual is infeasible.

2. Consider the following LP,

$$\text{Maximize } Z = -x_1 + 5x_2$$

subject to constraints

$$x_1 + 2x_2 \leq 0.5$$

$$-x_1 + 3x_2 \leq 0.5$$

and $x_1, x_2 \geq 0$. Suppose that Row (0) of the optimal tableau for this LP reads

Basic Variable	Row	Z	x_1	x_2	s_1	s_2	RHS
Z	(0)	1	0	0	0.4	1.4	??

Without going through the simplex algorithm, find the optimal value of the LP.

3. Consider the LP problem:

$$\text{Maximize } Z = 3x_1 + 2x_2 + 5x_3$$

subject to constraints

$$\begin{aligned}x_1 + 2x_2 + x_3 &\leq 30 \\3x_1 + 2x_3 &\leq 60 \\x_1 + 4x_2 &\leq 20\end{aligned}$$

and $x_1, x_2, x_3 \geq 0$. Someone tells you that the optimal solution has either

$$\text{basic variables } \mathbf{x}_B = [s_1, x_3, s_3] \text{ and } \mathbf{B}^{-1} = \begin{bmatrix} 1 & -0.5 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

or

$$\text{basic variables } \mathbf{x}_B = [x_3, s_2, s_3] \text{ and } \mathbf{B}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

but he cannot remember which.

- Without going through simplex algorithm, identify the correct set of basic variables for the optimal solution.
- Reconstruct the whole optimal table. From the optimal table, argue that the LP is degenerate.
- Suppose that the first constraint in the LP is now

$$x_1 + 2x_2 + x_3 \leq R.$$

For what range of R the basic variable \mathbf{x}_B is still optimal (i.e. \mathbf{x}_B is still the basic variables in the optimal solution)?