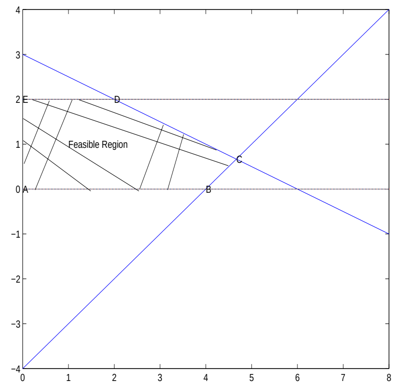
Solns to 2nd Homework

2 (a) the feasible region is the shaded part. And BFS are identified by A, B, C, D, E which are vetex points of the feasible region.



vertice:$A(0,0), B(0,2), C(2,2), D(14/3,2/3), E(4,0)$.

(b) Canonical form of constraint.

$$\begin{aligned} x_1 + 2x_2 + s_1 &= 6 \\ x_1 - x_2 + s_2 &= 4 \\ x_2 + s_3 &= 2 \end{aligned}$$

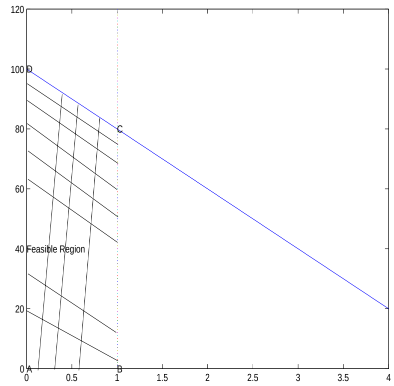
with $x_j \geq 0, j = 1,2,3$. $s_j \geq 0, j = 1,2$.

CPF	corresponding BFS
(0, 0)	(0, 0, 6, 4, 2)
(4, 0)	(4, 0, 2, 0, 2)
$(14/3, 2/3)$	$(14/3, 2/3, 0, 0, 4/3)$
(2, 2)	(2, 2, 0, 4, 2)
(0, 2)	(0, 2, 2, 6, 0)

(c) from the simplex table, we see the sequence is

(d) replace the optimal solution into constraints, we know that constraint 1, and 2 are tight constraint, the solution makes equal. And constraint 3 is a slack constraint.

3 (a) The vertices are:(0, 0),(1, 0),(1, 80),(0, 100) by graphical way.From these vertices, we get the corresponding BFS



CPF	corresponding BFS
(0, 0)	(0, 0, 1, 100)
(1, 0)	(1, 0, 0, 80)
(1, 80)	(1, 80, 0, 0)
(0, 100)	(0, 100, 1, 0)

(b) Do the simplex and find that the iteration goes this way: A,B,C,D in order.
iteration(0)

Basic Variable	Row	Z	X_1	X_2	S_1	S_2	RHS	Ratios
Z	0	1	-10	-1	0	0	0	
s_1	1	0	1	0	1	0	1	1
s_2	2	0	20	1	0	1	100	5

iteration(1)

Basic Variable	Row	Z	X_1	X_2	S_1	S_2	RHS	Ratios
Z	0	1	0	-1	10	0	10	
x_1	1	0	1	0	1	0	1	NA
s_2	2	0	0	1	-20	1	80	80

iteration (2)

Basic Variable	Row	Z	X_1	X_2	S_1	S_2	RHS	Ratios
Z	0	1	0	0	-10	1	90	
x_1	1	0	1	0	1	0	1	
x_2	2	0	0	1	-20	1	80	NA

iteration (3)

Basic Variable	Row	Z	X_1	X_2	S_1	S_2	RHS	Ratios
Z	0	1	10	0	0	1	100	
s_1	1	0	1	0	1	0	1	
x_2	2	0	20	1	0	1	100	

4 The canonical form of this lP problem is:

$$
\begin{aligned}
Z - 5x_1 + x_2 &= 0 \\
x_1 - 3x_2 + s_1 &= 1 \\
x_1 - 4x_2 + s_2 &= 3
\end{aligned}
$$

with $x_j \geq 0, j = 1, 2$. $s_j \geq 0, j = 1, 2$.
See the simplex form:

iteration(0)

Basic Varible	Row	Z	X_1	X_2	S_1	S_2	RHS	Ratios
Z	0	1	-5	1	0	0	0	
s_1	1	0	1	-3	1	0	1	1
s_2	2	0	1	-4	0	1	3	3

iteration (1)

Basic Varible	Row	Z	X_1	X_2	S_1	S_2	RHS	Ratios
Z	0	1	0	-14	5	0	0	5
x_1	1	0	1	-3	1	0	1	
s_2	2	0	0	-1	-1	1	2	

Note that the candidate entering variable is x_2, but there is no constraint on the entering value, this means the entering value can be any number we want. So the LP problem is unbounded.

5 Canonical form of this LP problem:

$$Z = 4x_1 + 3x_2 + 6x_3$$

subject to:

$$\begin{aligned} 3x_1 + x_2 + 3x_3 + s_1 &= 30 \\ 2x_1 + 2x_2 + 3x_3 + s_2 &= 30 \end{aligned}$$

with $x_j \geq 0, j = 1, 2, 3$. $s_j \geq 0, j = 1, 2$.
Initialization:First let $x_1, x_2 x_3$ be 0, then

$$\begin{aligned} Z - 4x_1 - 3x_2 - 6x_3 &= 0 \\ 3x_1 + x_2 + 3x_3 + s_1 &= 30 \\ 2x_1 + 2x_2 + 3x_3 + s_2 &= 40 \end{aligned}$$

Deciding entering variable: See the coefficient of x_3 is -6,which is the negative max, thus x_3 is the entering variable.
Deciding the leaving variable: see

$$\begin{aligned} 3x_3 \leq 30, x_1 \leq 10 \\ 3x_3 \leq 40, x_2 \leq 40/3 \end{aligned}$$

thus the leaving variable is s_1.
Gauss elimination: Result is

$$\begin{aligned} Z + 2x_1 - x_2 + 2s_1 &= 60 \\ x_1 + 1/3x_2 + x_3 + 1/3s_1 &= 10 \\ -x_1 + x_2 - s_1 + s_2 &= 10 \end{aligned}$$

Deciding entering variable: See now the coefficient of x_2 is -1,which is the negative max, thus x_2 is the entering variable.

Deciding the leaving variable: see

$$1/3x_2 \leq 10, x_2 \leq 30$$
$$x_2 \leq 10, x_2 \leq 10$$

thus the leaving variable is s_2.
Gauss elimination: Result is

$$\begin{aligned} Z + x_1 + s_1 + 2s_2 &= 70 \\ 4/3x_1 + x_3 + 2/3s_1 - 1/3s_2 &= 20/3 \\ -x_1 + x_2 - s_1 + s_2 &= 10 \end{aligned}$$

Thus the max z=70 when $x_1 = 0, s_1 = 0, s_2 = 0, x_2 = 10 from the sencond equation and x_3 = 20/3 from the first equation.$

6 The canonical form is

$$maximize Z = 2x_1 + 3x_2 - Ms_2$$

subject to:

$$\begin{aligned} x_1 + 2x_2 + s_1 &= 4 \\ x_1 + x_2 + s_2 &= 3 \end{aligned}$$

with $x_j \geq 0, j = 1,2,\ s_j \geq 0, j = 1,2.$
The simplex table goes

Basic Varible	Row	Z	X_1	X_2	S_1	S_2	RHS	Ratios
Z	0	1	-2	-3	0	M	0	
s_1	1	0	1	2	1	0	4	
s_2	2	0	1	1	0	1	3	

after Gauss elimination:

Basic Varible	Row	Z	X_1	X_2	S_1	S_2	RHS	Ratios
Z	0	1	-M-2	-M-3	0	0	-3M	
s_1	1	0	1	2	1	0	4	2
s_2	2	0	1	1	0	1	3	3

iteration(0)

Basic Varible	Row	Z	X_1	X_2	S_1	S_2	RHS	Ratios
Z	0	1	-M-2	0	(M+3)/2	0	6-M	
x_2	1	0	1/2	1	1/2	0	2	4
s_2	2	0	1/2	0	-1/2	1	1	2

iteration(01)

Basic Varible	Row	Z	X_1	X_2	S_1	S_2	RHS	Ratios
Z	0	1	0	0	1	M+1	7	
x_2	1	0	0	1	0	-1	1	
x_1	2	0	1	0	-1	2	2	

Thus the maximal $Z = 7$, when $x_1 = 2, x_2 = 1$.

7 Canonical form for the first phase:

$$maximize - Z = -a_1 - a_2$$

with constraints

$$\begin{aligned} x_1 + 4x_2 + 2x_3 - s_1 + a_1 &= 8 \\ 3x_1 + 2x_2 - s_2 + a_2 &= 6 \end{aligned}$$

The simplex table for this phase is:

Basic Varible	Row	Z	X_1	X_2	S_1	S_2	RHS	Ratios			
Z	0	1	0	0	0	0	0	1	1	0	
a_1	1	0	1	4	2	-1	0	1	0	8	
a_2	2	0	3	2	0	0	-1	0	1	6	

after Gauss elimination:

Basic Varible	Row	Z	X_1	X_2	S_1	S_2	a_1	a_2	RHS	Ratios	
Z	0	1	-4	-6	-2	1	1	0	0	-14	
a_1	1	0	1	4	2	-1	0	1	0	8	2
a_2	2	0	3	2	0	0	-1	0	1	6	3

iteration (0)

Basic Varible	Row	Z	X_1	X_2	S_1	S_2	a_1	a_2	RHS	Ratios	
Z	0	1	-5/2	0	1	-1/2	1	3/2	0	-2	
x_2	1	0	1/4	1	1/2	-1/4	0	1/4	0	2	8
a_2	2	0	5/2	0	-1	1/2	-1	-1/2	1	2	8/5

iteration(1)

Basic Varible	Row	Z	X_1	X_2	S_1	S_2	a_1	a_2	RHS	Ratios	
Z	0	1	0	0	0	0	0	1	1	0	
x_2	1	0	0	1	6/10	-6/20	1/10	6/20	-1/10	9/5	
x_1	2	0	1	0	-2/5	1/5	-2/5	-1/5	2/5	4/5	

see there is no negative number in the first line, thus go to phase 2. Drop the artificial variable, we get

Basic Varible	Row	Z	X_1	X_2	S_1	S_2	RHS	Ratios	
Z	0	1	2	3	1	0	0	0	
x_2	1	0	0	1	6/10	-6/20	1/10	9/5	
x_1	2	0	1	0	-2/5	1/5	-2/5	4/5	

after Gauss elimination

Basic Varible	Row	Z	X_1	X_2	S_1	S_2	RHS	Ratios	
Z	0	1	0	0	0	1/2	5/10	-7	
x_2	1	0	0	1	6/10	-6/20	1/10	9/5	
x_1	2	0	1	0	-2/5	1/5	-2/5	4/5	

Thus we know max $-Z = -7$, which means min$Z = 7$, when $x_1 = 4/5, x_2 = 9/5, x_3 = 0$.