AM 121: Homework # 2 (Due Oct 11, Tuesday)

The book we refer to is Hillier & Lieberman, *Introduction to Operations Research* (8th Edition).

1. Read sections 4.1 to 4.6.

2. Consider the following LP:

$$\text{Maximize} \quad Z = x_1$$

   subject to constraints

$$
\begin{array}{lrcl}
(1) & x_1 + 2x_2 & \leq & 6 \\
(2) & x_1 - x_2 & \leq & 4 \\
(3) & x_2 & \leq & 2
\end{array}
$$

   and $x_1, x_2 \geq 0$.

   (a) Draw the feasible region in (x_1, x_2)-space, and identify all the vertice.

   (b) Transform the LP into canonical form by adding slack variables. Find all the BFS. For each vertex in (a), find the corresponding BFS.

   (c) Use simplex algorithm in tabular form to solve the LP. Describe graphically the sequence of vertice that the simplex algorithm has scanned.

   (d) Identify all the tight constraints by looking at the optimal solution, and verify this graphically.

3. Simplex algorithm is usually very efficient. It only need to scan a very small number of the vertices (or, BFS) to reach an optimal solution. Consider the following LP:

$$\text{Maximize} \quad Z = 10x_1 + x_2$$

   subject to constraints

$$
\begin{array}{rcl}
x_1 & \leq & 1 \\
20x_1 + x_2 & \leq & 100
\end{array}
$$

   and $x_1, x_2 \geq 0$.

   (a) Find all the BFS for this LP.

   (b) Perform the simplex algorithm to solve this LP. Observe that every BFS will be examined before the optimal solution is found.

   Something about history: By generalizing this example, Klee and Minty (1972) constructed (for $n = 2, 3, \cdots$) an LP with n decision variables and n constraints for which the simplex algorithm examines $2^n - 1$ BFS before the optimal solution is found. Fortunately, such pathological LPs rarely occur in practice.

4. Consider the following LP:

$$\text{Maximize} \qquad Z = 5x_1 - x_2$$

subject to constraints

$$\begin{aligned} x_1 - 3x_2 &\leq 1 \\ x_1 - 4x_2 &\leq 3 \end{aligned}$$

and $x_1, x_2 \geq 0$. Use simplex algorithm to show that this LP is unbounded.

5. Solve problem 4.3-3.

6. Solve Problem 4.6-1. Ignore the questions (a)(b)(c). Just use Big-M method to find the optimal solution.

7. Solve Problem 4.6-3. Ignore the questions (a)(b)(c)(d)(e). Just use the Two-Phase method to find the optimal solution.

IN ORDER TO SAVE THE TROUBLE OF DRAWING UP THE TABLES, YOU CAN USE THE TABLES IN THE NEXT A FEW PAGES. YOU CAN USE THESE TABLES IN YOUR HOMEWORK. IF YOU NEED MORE, PHOTOCOPY THEM.

Basic Variable	Row	Z		RHS	Ratios

Basic Variable	Row	Z		RHS	Ratios

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