

## AM 121: Homework # 2 (Due Oct 5, Tuesday)

The book we refer to is Hillier & Lieberman, *Introduction to Operations Research* (8th Edition).

1. Read sections 4.1 to 4.6.
2. Consider the following LP:

$$\text{Maximize } Z = x_1$$

subject to constraints

$$\begin{aligned} (1) \quad & x_1 + 2x_2 \leq 6 \\ (2) \quad & x_1 - x_2 \leq 4 \\ (3) \quad & x_2 \leq 2 \end{aligned}$$

and  $x_1, x_2 \geq 0$ .

- (a) Draw the feasible region in  $(x_1, x_2)$ -space, and identify all the vertices.
  - (b) Transform the LP into canonical form by adding slack variables. Find all the BFS. For each vertex in (a), find the corresponding BFS.
  - (c) Use simplex algorithm in tabular form to solve the LP. Describe graphically the sequence of vertices that the simplex algorithm has scanned.
  - (d) Identify all the tight constraints by looking at the optimal solution, and verify this graphically.
3. Simplex algorithm is usually very efficient. It only needs to scan a very small number of the vertices (or, BFS) to reach an optimal solution. Consider the following LP:

$$\text{Maximize } Z = 10x_1 + x_2$$

subject to constraints

$$\begin{aligned} x_1 &\leq 1 \\ 20x_1 + x_2 &\leq 100 \end{aligned}$$

and  $x_1, x_2 \geq 0$ .

- (a) Find all the BFS for this LP.
- (b) Perform the simplex algorithm to solve this LP. Observe that every BFS will be examined before the optimal solution is found.

*Something about history:* By generalizing this example, Klee and Minty (1972) constructed (for  $n = 2, 3, \dots$ ) an LP with  $n$  decision variables and  $n$  constraints for which the simplex algorithm examines  $2^n - 1$  BFS before the optimal solution is found. Fortunately, such pathological LPs rarely occur in practice.

4. Consider the following LP:

$$\text{Maximize } Z = 5x_1 - x_2$$

subject to constraints

$$x_1 - 3x_2 \leq 1$$

$$x_1 - 4x_2 \leq 3$$

and  $x_1, x_2 \geq 0$ . Use simplex algorithm to show that this LP is unbounded.

5. Solve problem 4.3-3.

6. Solve Problem 4.6-1. Ignore the questions (a)(b)(c). Just use Big-M method to find the optimal solution.

7. Solve Problem 4.6-3. Ignore the questions (a)(b)(c)(d)(e). Just use the Two-Phase method to find the optimal solution.

IN ORDER TO SAVE THE TROUBLE OF DRAWING UP THE TABLES, YOU CAN USE THE TABLES IN THE NEXT A FEW PAGES. YOU CAN USE THESE TABLES IN YOUR HOMEWORK. IF YOU NEED MORE, PHOTOCOPY THEM.

Basic Variable	Row	$Z$		RHS	Ratios

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