GRAVITY EQUATIONS

Yes, they are not very pretty, but they deserve a few comments:

- They really are different from one another.
- *Caution*: Γ often includes undisplayed references to boundary data that appear in $\nabla F = \nabla H - \nabla B$. The origin of the z-coordinate cannot matter, so values of B and H appear only in $F = H - B$.
- The gravity equation is the essential mechanism by which *dispersion* is introduced into these shallow water approximations. Without it, when $G = g$, waves almost always steepen to become N-waves or bores; with it, there are also cnoidal waves, solitary waves and undular jumps.
- The results show how the pressure boundary conditions can be absorbed in the gravity equation. The boundary conditions on $\grave{p}$ are $\grave{p}_H = \grave{p}_B = 0$, so $\grave{p}$ is determined by a nonhomogeneous Dirichlet problem with homogeneous boundary data.
- Since $\grave{p}$ can be found by solving a Poisson equation, it follows that there are no more treacherous, *hidden* time derivatives. Approximations that take $\grave{p}$ into consideration are more accurate, but not qualitatively different.
- With respect to G, the gravity equation is $\mathcal{L}G = g + \Gamma$, where $\mathcal{L}$ is a linear operator, and the coefficient of G is usually positive. If the coefficient becomes *small* in a steepening beach wave, the steepening is counteracted by the the largeness of G in the horizontal momentum equations. I don't know if that keeps the coefficient from ever changing sign.
- When $1 + (\nabla H)^\top \nabla B \to 0$ in the runup of an unbroken wave (i.e. the surface and the beach are nearly perpendicular where $F \to 0$) G becomes quite large, and the near impulse causes a jet-like, accelerated runup of the foot of the wave. It is the shallow water version of the *shoreline singularity*. If the runback of the previous wave is too strong, the incoming one breaks before the impulse can happen.

- In *runback* $(\nabla H)^\top \nabla B$ is positive, and the reduced effective gravity causes a noticeably reduced acceleration of the falling swashline.

- The *other* term in the coefficient of G becomes infinite (and positive) at the point on a plunging breaker where the surface turns the corner, and that brings G to *zero* there. In the upper layer, bounded above and below by air, $p_B = p_H$, G is *zero*, horizontal accelerations are *zero*, regardless of ∇H, and the vertical acceleration is $-g$. The upper layer has two more degrees of freedom ($DW/Dt + \cdots$ and $DZ/Dt + \cdots$), and the motion of the upper layer is essentially that of a deformable projectile.

  (Of course, the detailed behavior at the swashline and in breakers depends upon *many* other effects, but at least these results appear to be consistent with observations I have made on beaches all over the world.)

- The results of first order approximations are not qualitatively different from second order results, and much of this could have been inferred from the results of Green and Naghdi.