**The Saddle-Node Bifurcation**

Consider the autonomous system of differential equations

 $\begin{matrix}\frac{dx}{dt}=x^{2}-μ\\\frac{dy}{dt}=-y\end{matrix}$

where ** is a parameter.

**If ** > 0:**

There are two equilibria: (–√**, 0) and (√**, 0).

The Jacobian at (–√**, 0) is: $\left[\begin{matrix}-2\sqrt{μ}&0\\0&-1\end{matrix}\right]$ .

Thus, *T* = – 2√** –1 and *D* = 2√**, yielding *T*2/4 – *D* = (2√**– )2/4 > 0. Hence (–√**, 0) is a stable node.

The Jacobian at (√**, 0) is: $\left[\begin{matrix}2\sqrt{μ}&0\\0&-1\end{matrix}\right]$ .



Thus, *T* = 2√** –1 and *D* = – 2√**. Hence (√**, 0) is a saddle point.

To the left is the phase portrait obtained for ** = 1, showing the two equilibria:

attracting node at (–1, 0),

saddle point at (1, 0).



**If ** < 0:**

There is no equilibrium.

To the left is the phase portrait obtained for ** = –1.

**Conclusion:**

A bifurcation takes place at ** = 0. This type of bifurcation, where two equilibria, a stable node and a saddle point, disappear by merging with each other is called a **saddle-node bifurcation**.

This type of bifurcation occurs in the excitatory-inhibitory network which we shall study next.