US Financial Debt Crisis: A Stochastic Optimal Control Approach

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Abstract: The financial crisis was precipitated by the mortgage crisis. A whole structure of financial derivatives was based upon the ultimate debtors, the mortgagors. Insofar as the mortgagors were unable to service their debts, the values of the derivatives fell. The financial intermediaries whose assets and liabilities were based upon the value of derivatives were very highly leveraged. Changes in the values of their net worth were large multiples of changes in asset values. A cascade was precipitated by the mortgage defaults. In this manner, the mortgage debt crisis turned into a financial crisis. The crucial variable is the optimal debt of the real estate sector, which depends upon the capital gain and the interest rate. I apply the Stochastic Optimal Control (SOC) analysis to derive the optimal debt. Two models of the stochastic process on the capital gain and interest rate are presented. Each implies a different value of the optimal debt/net worth. I derive an upper bound of the optimal debt ratio, based upon the alternative models. An empirical measure of the excess debt: actual less the upper bound of the optimal ratio, is shown to be an early warning signal (EWS) of the debt crisis.

Introduction

The mortgage/housing market had increased in importance in financial markets from 1985 to 2005. The assets of the banking sector and financial intermediaries were closely and significantly tied to the real estate market. Contrary to the Efficient Market Hypothesis, there was indeed a mortgage market bubble, and prices did not reflect publicly available information on fundamentals. I measure the "bubble" as an unsustainable debt/income ratio of households. The "Quants"[2] in Wall Street and the market were oblivious to systemic risk and ignored publicly available information concerning the debt ratios of the households. Using Stochastic Optimal Control (SOC) analysis, I derive an optimal debt ratio. An Early Warning Signal (EWS) of a debt crisis is the excess debt, defined as the difference between the actual and optimal debt ratio of households. The excess debt determines the probability of a debt crisis.

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[2] The "Quants" are physicists, computer scientists and mathematicians who practice financial engineering.

When the households were experiencing difficulties in servicing their debts, the highly leveraged banks and financial intermediaries suffered significant losses and their net worth declined to an alarming degree. This was the financial crisis of 2008. In the Savings and Loan and agricultural crises of the 1980s, the mortgages were not a very important part of portfolios of banks and financial intermediaries. Hence these earlier crises did not have significant effects upon the US financial system.

Part 1 concerns the importance of the housing sector to the financial sector. Part 2 concerns the characteristics of the mortgage market. The SOC analysis concerning the optimal debt ratio is in part 3. An economic interpretation of this is the subject of part 4. Part 5 concerns empirical measures of the optimal debt ratio. Early Warning Signals (EWS) of a crisis are presented in part 6. The model used by the market, which led to the crisis, is the subject of part 7. Part 8 describes the Shadow Banking System, leverage and financial linkages.

1. The importance of the housing/mortgage sector to the financial sector

Juselius and Kim examine different loan categories of banks, to explain financial fragility. The recent crisis was predominantly caused by too large financial obligations/mortages of the household sector to the banks and intermediaries, whereas the recession in the eighties was more related to the business sector.

Table 1 describes the changing composition of bank portfolios. Real estate loans rose from 30% in 1985q1 to 57% in 2005q1. Moreover, the share of households in the banks' real estate loans rose from 48% in 1985q1 to 58% in 2005q1.

Table 1. Banks' Aggregate Portfolio

	1985q1	1995q1	2001q1	2005q1
Share total loans				
(1)Real estate	30%	46%	47%	57%
(2) Business loans	34	23	26	20
(3) other	36	31	27	23
Share R.E. Loans				
(4) Households	48	59	56	58
(5) Business	26	29	28	26
(6) other	26	12	16	16

Source: Juselius and Kim (2011).

An important finding is that the time path of the loss rate on real estate loans resembles that of the bank failure rate to a much greater extent than does the loss rate on business loans. They infer that financial stability was more sensitive to real estate loans than to business loans in the recent period. They write: "… there are only two episodes of major bank failures in our sample. The first occurs between the late 80's and early 90's, and is associated with the savings and loan crisis, whereas the second corresponds to the recent financial crisis. It is notable that the bank failure rate is very low between these two periods, suggesting that the burst of the dot.com bubble had little effect on the incidence of bank failures".

2. Characteristics of the Mortgage Market

Demyanyk and Van Hemert (2008) utilized a data-base containing information on about one half of all subprime mortgages originated between 2001 and 2006. They explored to what extent the probability of delinquency/default can be attributed to different loan and borrower characteristics and housing price appreciation. Figure 1 plots Household debt service/disposable income. From 1998-2005 rising home prices produced above average capital gains, which increased owner equity. This induced a supply of mortgages, and the totality of household financial obligations as a percent of disposable personal income rose. The rises in house prices and owner equity induced a demand for mortgages by banks and funds.
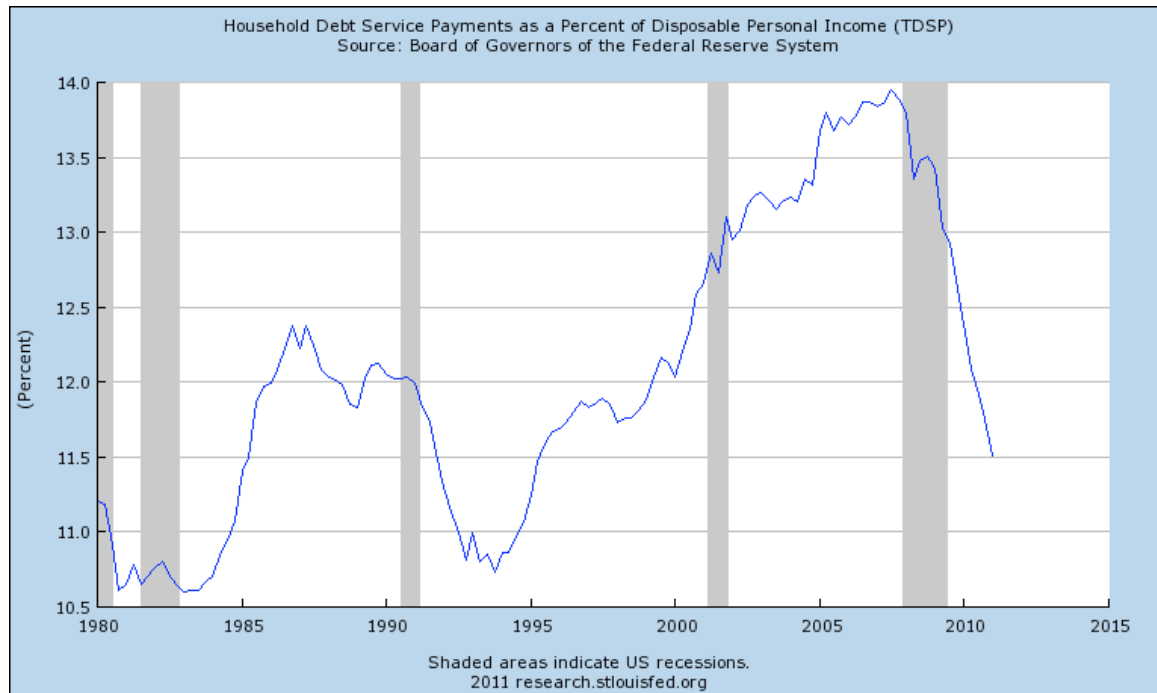
Figure 1. Household debt service payments as a percent of disposable personal income, 1980 – 2011. Source: Federal Reserve of St. Louis, FRED, from Federal Reserve

In about 45-55% of the cases, the purpose of the subprime mortgage taken out in 2006 was to extract cash by refinancing an existing mortgage loan into a larger mortgage loan. The mortgagor anticipated paying the interest on the larger loan from an anticipated capital gain. This was a "free lunch".

The quality of loans declined. The share of loans with full documentation substantially decreased from 69% in 2001 to 45% in 2006. Funds held packages of mortgage-backed securities either directly as asset-backed securities or indirectly through investment in central funds. The purchases were financed by short-term bank loans. Neither the funds nor the banks worried about the rising debt, because equity was rising due to the rise in home prices.

Figure 2 plots the capital gain and debt ratio. They are normalized so that each variable has a mean of zero and standard deviation of unity. The vertical axis plots a t-statistic. The figure shows that from 2003-2005 the capital gain was more than two standard deviations above its long term mean, and the debt ratio was about two standard deviations above its long term mean. Since the mean interest rate over the entire period was about equal to the mean capital gain, a value of $t = 2$ should have been a warning signal that the capital gain cannot continue to exceed the mean interest rate. The "free lunch could not continue. Once the capital gain fell below the interest rate, the debtors could not service their debts, and foreclosures and delinquencies would have led to the collapse of the value of the financial derivatives. The Quants ignored the economic structure of the mortgage market .
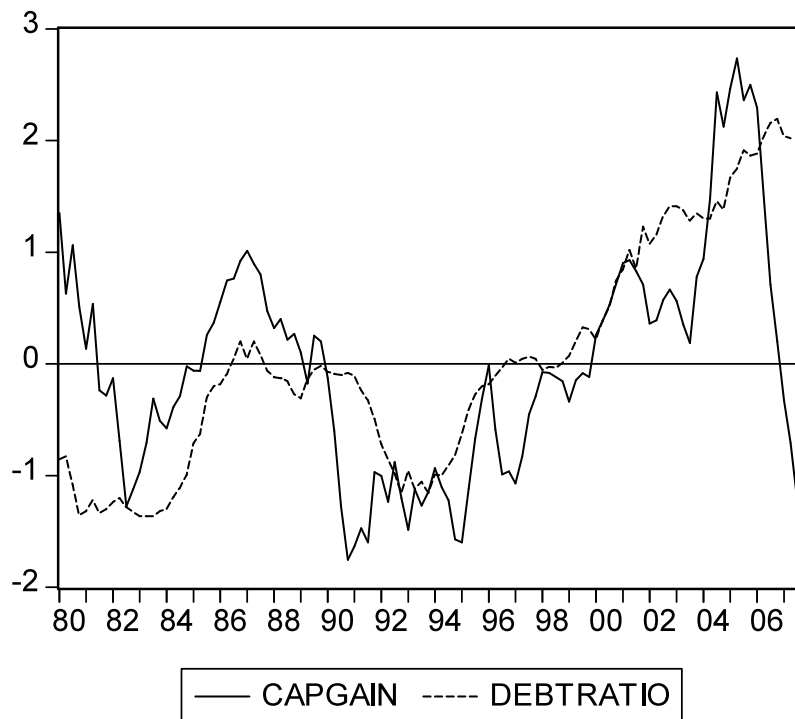
Figure 2. Mortgage Market Bubble. Normalized variables. Appreciation of single-family housing prices, CAPGAIN, 4q appreciation of US Housing prices HPI, Office Federal Housing Enterprise Oversight (OHEO); Household debt ratio DEBTRATIO = household financial obligations as a percent of disposable income. Federal Reserve Bank of St. Louis, FRED, Series FODSP. Sample 1980q1 – 2007q4.

This view is confirmed in the study by Demyanyk and Van Hemert who had a data base consisting of one half of the US subprime mortgages originated during the period 2001-2006. At every mortgage age, loans originating in 2006 had a higher delinquency rate than in all the other years since 2001. They examined the relation between the probability Π of delinquency/foreclosure/binary variable z, denoted as $\Pi = Pr(z)$ and sensible economic variables, vector X. They investigated to what extent a logit regression $\Pi = Pr((z)) = \Phi(\beta X)$ can explain the high level of delinquencies of vintage 2006 mortgage loans.

For year 2006, the largest contribution to delinquency and to foreclosure was the low house price appreciation. The absolute contributions are small, but the relative contributions are significant. The house price appreciation factor was 7 times the contributions of the debt/income and documentation ratios for the delinquency rate, and 15 times the contributions of the debt/income and documentation ratios for the foreclosure rate. The work of D-VH confirms the analysis above.

3. The Stochastic Optimal Control Analysis[3]

The financial structure rested upon the ability of the mortgagors to service their debts. That is where "systemic risk" was to be found. When the households could not service their debts, the highly leveraged financial structure collapsed. For this reason the focus is upon the optimal debt ratio of households.

The stochastic optimal control approach postulates a hypothetical/idealized/ optimizer investor in real estate who finances his purchase with a mortgage debt. The optimum debt ratio f(t) is selected to maximize the expectation of a concave function of net worth X(T) at some terminal date, equation (1), subject to stochastic processes on the house price and interest rate. This is a risk averse criterion since the logarithm is a highly concave function, where losses are more heavily weighted than equal gains.
(1) $\max_f E[\ln X(T)]$.

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[3] A mathematical analysis is in Fleming and Stein (2004), Stein (2006), (2011). The exposition in this article is more graphic and intuitive.

The optimum debt/net worth ratio depends most crucially upon the assumed stochastic process concerning the price or capital gains variable. The expected growth of net worth is maximal when the debt ratio is optimal. As the debt ratio exceeds the optimal, the expected growth declines and the variance of growth increases. The excess debt is the actual less the optimal. As the excess debt rises, the probability that the household cannot pay its debt increases. Thereby we have an early warning signal of a crisis.

Since no one knows what is the "true" stochastic process there is uncertainty concerning the true optimal debt ratio. The difference between the actual debt ratio and the true optimal is directly related to the difference between the actual growth of net worth and the optimal growth. This way one can have bounds for the effect of misspecification of stochastic process upon growth of net worth.

Several stochastic processes are considered and the optimal debt ratio is derived for each case. In this article, I simply state the optimal debt ratio that is implied by the alternative stochastic processes, Model I and Model II. On the basis of models I/II, I derive an equation for an empirical estimate of the optimal debt ratio. That is the benchmark optimal ratio. I then derive empirical estimates of the excess debt and show that it is a warning signal of a crisis. In this manner, I reject the Greenspan, Federal Reserve, Treasury et al view that the crisis was unpredictable.

Net worth $X(t)$ is assets $A(t)$ less liabilities $L(t)$. Assets equals net worth plus debt. Equation (2) or (2a) is the stochastic differential equation for the growth of net worth, the change in assets less the change in liabilities. The change in net worth is the sum of several terms. The first term is assets times its return. The return consists of the capital gain plus the productivity of the assets. The second term is the debt payments, the interest rate $i(t)$ times $L(t)$ the debt. The last term is consumption or dividends. Let $f(t) = L(t)/X(t)$ denote the ratio of debt/net worth. The capital gain is $dP(t)/P(t)$ where $P(t)$ represents an index of house prices. The productivity of capital $\beta(t)$ is income/assets, which is deterministic or constant; and c is the constant ratio of consumption or dividends/net worth. The change in net worth is stochastic differential equation (2). Capital gain and interest rate are stochastic in the models.

(2a) Change in net worth = assets (capital gain + productivity of assets) – (interest rate) (debt) – consumption.

(2) dX(t) = X(t)[(1 + f(t)) (dP(t)/P(t) +β(t) dt) - i(t)f(t) – c dt]

The optimal debt ratio is the *control variable* that maximizes the *criterion function (1)*, the expected logarithm of net worth, subject to the *stochastic differential equation* (2). The main uncertainty is in the capital gain term dP(t)/P(t), which is unknown when the debt ratio is selected. I also allow uncertainty about the interest rate. The models discussed here differ in the assumptions concerning the stochastic process on the capital gains term. Hence there are differences in what is the optimal debt ratio.

Models I and II are presented and discussed. I then derive empirical measures of the optimal debt ratio appropriate for each model. The next step is to compare the actual f(t) to the derived optimal debt f*(t). In section 5, I derive an upper bound f** > f* for both ratios to calculate the excess debt Ψ(t) = f(t) – f**(t), which is a warning signal of a 2007-08 crisis. In section 6, I explained why the actual debt ratio deviated from the optimal, and led to the crisis.

Model I

In model I there are two sources of uncertainty: the price of the asset and the interest rate. Equations (3) – (4) concern the price P(t) of the asset. The productivity of capital β(t) = β can be viewed as deterministic or constant and consumption ratio c(t) = c is constant. Model I contains two ideas, inspired by Bielecki and Pliska (1999) and Platen-Rebolledo (1995), and discussed in Fleming (1999). Equation (3) states that the price consists of two components: a trend and a deviation from it. The price trend is r. The initial value of the price is P = 1. The second component y(t) is a deviation from the trend. The second idea, expressed in equations (4)-(5), is that deviation y(t) is an ergodic mean reversion term whereby the price converges towards the trend. The speed of convergence of the deviation y(t) towards the trend is described by finite coefficient $\alpha > 0$. The stochastic term is $\sigma_p \, dw_p(t)$. The solution of stochastic differential equation

(4) is (5). The deviation from trend converges to a distribution with a mean of zero and a variance of $\sigma^2/2\alpha$.

(3) $P(t) = P \exp(rt + y(t))$, $P = 1$,

(3a) $y(t) = \ln P(t) - \ln P - rt$.

(4) $dy(t) = -\alpha y(t)dt + \sigma_p \, dw_p(t)$. $\infty > \alpha > 0$, $E(dw) = 0$, $E(dw)^2 = dt$.

(5) $\lim y(t) \sim N(0, \sigma^2/2\alpha)$.

The choice of price trend r is very important in determining the optimal leverage. I impose a constraint that the assumed price trend must not exceed the mean rate of interest. If this constraint is violated, as occurred during the housing price bubble, debtors were offered a "free lunch" as described above. Borrow/Refinance the house and incur a debt that grows at i, the mean rate of interest. Spend the money in any way that one chooses. Insofar as the house appreciates at a rate greater than the mean rate of interest, at the terminal date T the house is worth more than the value of the loan, $P(T) > L(T)$. The debt $L(T)$ is easily repaid by selling the house at $P(T)$ or refinancing. One has had a free lunch. In the optimization, one must constrain the trend r not to exceed the *mean* rate of interest i(t). This constraint is equation (6).

(6) $r \leq i$.   No free lunch constraint

An alternative justification for equation (6) is as follows. The present value PV of the asset

(6a) $PV(T) = P(0) \exp[(r - i)t]$,

where trend r is the rate of appreciation or capital gain and i is the mean interest rate. If $(r - i) > 0$, the present value diverges to plus infinity. An infinite present value is not sustainable. The real rate of interest is (i-r), which should be non-negative.

Equation (7) concerns the interest rate uncertainty. The interest rate i(t) is the sum of a constant i dt plus a Brownian Motion (BM) term $\sigma_i \, dw_i(t)$. The two BM terms in equations (4) and (7) are correlated $0 > \rho \geq -1$, where the correlation ρ is presumed to be negative. When interest rates rise (decline) there are capital losses (gains). This negative correlation was an important factor in the house price bubble.

(7) $i(t) = i \, dt + \sigma_i \, dw_i(t)$.

The optimal debt ratio in Model I is equation (8), BOX 1. It is the solution of equation (1) subject to equations (2), (3) and (4). It states that the optimal debt ratio f*(t), that maximizes the expected growth of net worth, is positively related to the productivity of capital less the real rate of interest $[\beta - (i - r)]$ = Net Return, and negatively related to the deviation of the price from trend $\alpha y(t)$ and negatively related to risk terms contained in σ^2 equation (8a) . The negative correlation $\rho < 0$ between the capital gain and the interest rate increases the risk σ^2 and reduces the optimal ratio. Equation (8b) expresses in words the content of (8)-(8a).

Model II

Fleming and Stein call Model II "A Prototype Model". The mathematical solution[4] is in Fleming and Stein (2004) and Stein (2006), (2011). The stochastic variables in equation (2) and model II are: (i) the capital gain dP(t)/P(t) and (ii) the interest rate i(t). The capital gain has a drift of $\pi\,dt$ and a diffusion of $\sigma_p dw_{p.}$ equation (9). The productivity of capital $\beta(t)$ is deterministic or constant. The interest rate in equation (10) has a mean of i dt and the BM term is $\sigma_i dw_i$ whose expectation is zero.

(9) $dP(t)/P(t) + \beta(t) = \pi\,dt + \beta(t)\,dt + \sigma_p dw_p$

(10) $i(t) = i\,dt + \sigma_i dw_i.$

As in Model I, the two BM terms are expected to be negatively correlated, $\rho\,dt = E(w_b dw_i) < 0$. When the interest rate declines (rises), there are the capital gains (losses).

The optimal debt/net worth f*(t) is equation (11), where the optimal ratio is f*, a constant. The variance or risk is equation (11b) in BOX 1. The NET RETURN $=[\pi + (\beta - i)]$ is the productivity of capital plus *drift* of capital gain less the *mean* rate of interest. The debt ratio is negatively related to risk terms contained in σ^2 equation (11b) . The negative correlation $\rho < 0$ between the capital gain and the interest rate increases the risk σ^2 equation (11b) and reduces the optimal ratio.

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[4] Fleming and Stein use a HARA criterion function and use dynamic programming in the solution. Stein uses the logarithmic criterion function and Ito equation.

BOX 1 Optimum Debt/Net worth Ratio f*(t)

Model I

(8) $f^*(t) = \{[\beta - (i - r)] - (1/2) \sigma_p^2 - \alpha y(t) + \rho\sigma_i\sigma_p\}/ \sigma^2$

(8a) $\sigma^2 = \sigma_p^2 + \sigma_y^2 - 2 \rho\sigma_i\sigma_p$ $\rho < 0$

Net return $N = \{[\beta - (i - r)] - (1/2) \sigma_p^2 - \alpha y(t)\}$, $RISK = \sigma^2$

(8b)   Optimal debt ratio $= [\text{Net Return - Risk elements}]/\text{Risk}$

Model II

(11) $f^*(t) = f^* = [\beta + (\pi - i) - (\sigma_p^2 - \rho\sigma_i\sigma_p)]/ \sigma^2$

(11b) $\sigma^2 = \sigma_i^2 + \sigma_p^2 - 2\rho\sigma_i\sigma_p$ $= \text{Risk}$

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4. Interpretation of Optimal Debt Ratio

Figure 3 describes the optimal debt ratio f*(t) in a manner that is valid for either model. The optimal debt ratio is on the vertical axis. It is equation (8) for Model I and equation (11) for Model II. The horizontal axis measures the Net Return. The optimal debt ratio is a linear function of the Net Return. The risk premium is R in figure 3. The optimal debt ratio is positive only if the Net Return exceeds the risk premium.

The SOC analysis in Stein (2006), (2011) proved that the expected growth of net worth is maximal along the "optimal debt ratio line". This line relates the optimal debt ratio to the net return. The optimal debt ratio is not a constant, but varies directly with the net return and risk. If the Net Return is N(1), the optimal debt ratio is f*(1).
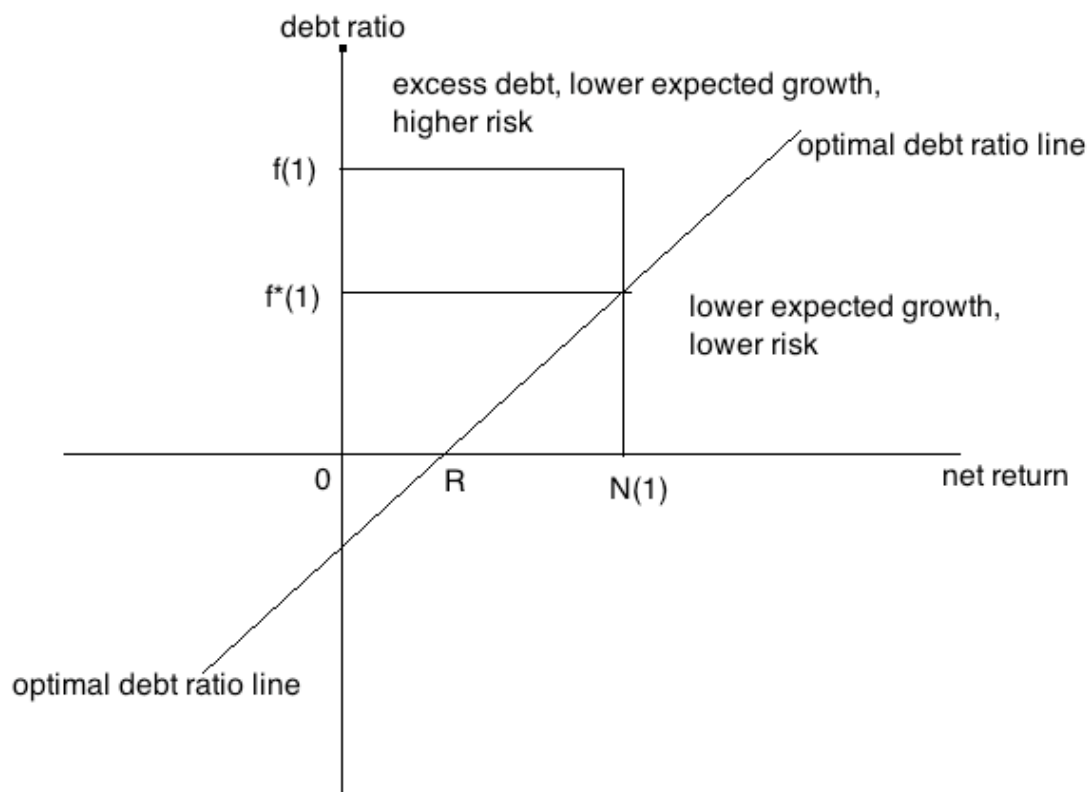
Figure 3. The optimal ratio f(t) of debt/net worth

As the actual debt ratio f(1) rises above the optimal debt ratio the line, the expected growth declines and the risk rises. If the actual debt ratio is f(1) and the net return is N(1), there is an excess debt Ψ(t) = f(1) – f*(1) > 0 and the expected growth is less than maximal. The larger is the excess debt, the lower the expected growth, the greater is the variance and hence the greater the probability that net worth will be driven towards zero. An early warning signal of a crisis is an excess debt. *The main conclusion is that it is not the debt per se that leads to a crisis, but it is the excess debt.*

5. Empirical Measures of an upper bound of the Optimal and actual debt ratio

The *optimum debt ratio* f* is based upon the equations in BOX 1, with the constraint that the *mean* real interest rate (i – r) is non negative – to avoid the "free lunch" difficulty. There are other sensible stochastic processes. No one knows what is the correct way to model the stochastic processes. My strategy is to derive an upper bound for the optimal debt ratio f*(t) in BOX 1 which will also be compatible with alternative sensible stochastic processes. Call the upper bound ratio f**. Then the excess debt Ψ(t) = f(t) – f**(t) is measured as the actual debt ratio less an upper bound f** ratio. The most problematic issue is how to measure the stochastic price, the capital gain dP(t)/P(t).

From the histogram of the capital gains in figure 4, the mean capital gain over the period 1980q1 – 2007q4 was 5.4% per annum with a standard deviation of 2.9%. There were no subperiods of price decline, and there was a large positive tail. Mortgages were considered to be safe assets.
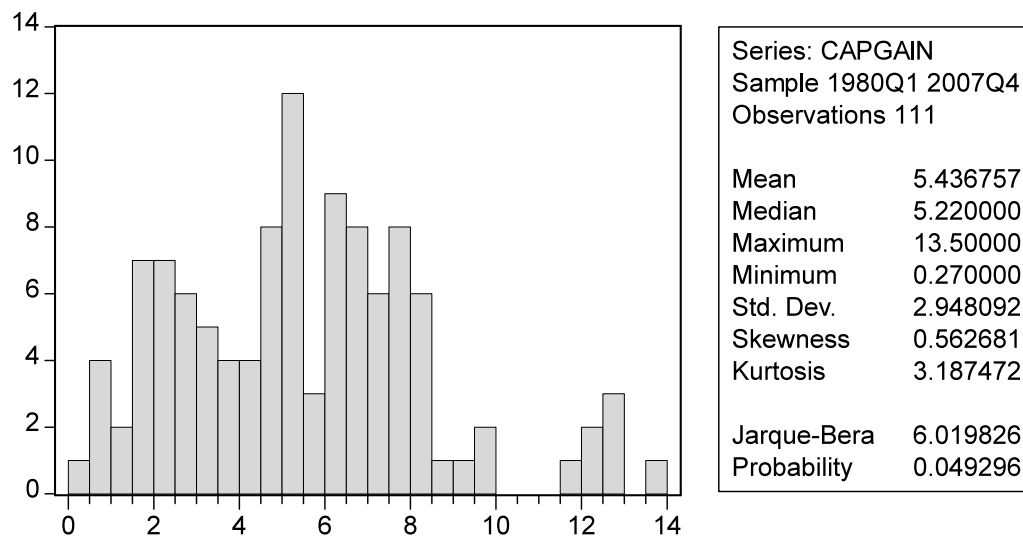
Figure 4. Histogram and statistics of CAPGAINS = Housing Price Appreciation
HPA, the change from previous 4-quarter appreciation of US housing prices,
percent/year, on horizontal axis. Frequency is on vertical axis.

The 30-year conventional mortgage rate ranged from 7.4% to 5 % pa from 2002-07. It is reasonable to argue that, over a long period, the appreciation of housing prices was not significantly different from "the mean mortgage rate of interest". Define $(i-r)$ as the mean interest rate less the trend growth of price, called the mean real rate of interest. A value of $(i-r) \geq 0$, a non-negative mean real rate of interest, precludes the "free lunch" that characterized the bubble. Then one component of an upper bound $f^{**}(t)$ in Model I eqn. (8) for the optimal debt ratio is $(i-r) = 0$. The other variable is $y(t)$ the deviation of price from trend. Since it is difficult to estimate the appropriate trend or price drift, I assume that deviation from trend $y(t) = 0$. Using these two assumptions in equation (8) an upper bound of the optimal debt ratio $f^{**}(t)$ is equation (12). We take $\beta(t)$ to be deterministic or a constant.

(12) $f^{**}(t) = \{[\beta(t) - (1/2)\, \sigma_p^2 + \rho\sigma_i\sigma_p]/\, \sigma^2$ $y(t) = 0,\ (i - r) = 0$

In Model II, the corresponding assumption is that the mean drift of the price is equal to the mean rate of interest, $\pi = i$. Then $f^{**}(t)$ for model II is (13).

(13) $f^{**}(t) = [\, \beta(t) - (\sigma_p^2 - \rho\sigma_i\sigma_p)]/\, \sigma^2$

$f^{**}(t) = [(R(t)/P(t)) - (risk\ elements\,)]/\sigma^2 > f^*(t)$

(13a)   *Mean $f^{**}(t) = [mean\ R(t)/P(t) - (risk\ elements\,)]/\sigma^2$*

We must estimate $\beta(t)$, the productivity of capital. The productivity of housing capital is the implicit net rental income/value of the home plus a convenience yield in owning one's home. Assume that the convenience yield in owning a home has been relatively constant. The productivity of capital is rental income $R(t)$ divided by the value of housing $P(t)Q(t)$, where $P(t)$ is an index of house prices and $Q(t)$ is an index of the "quantity" of housing. Thus $\beta(t) = R(t)/P(t)Q(t)$. I approximate the return $\beta(t)$ by using the ratio $R(t)/P(t)$ of rental income/an index of house prices. This is equation (13), whose mean is (13a).

The units of $R(t)$ and $P(t)$ are different. The numerator is in dollars and the denominator is a price index. In order to make alterative measures of the debt ratio and key economic variables comparable, I use normalized variables where the normalization

(N) of a variable Z(t) called N(Z) = [Z(t) – mean Z]/standard deviation. The normalized f**(t) is basically (13) minus (13a). The risk elements in the numerator cancel.

The mean of N(Z) is zero and its standard deviation is unity. The normalized upper bound f**(t) of optimal debt ratio is (14) or (14a). Call (14b) the RENTPRICE. It is the (rental income/index of house prices – mean)/standard deviation. The risk elements in the numerator of (13) cancel, in deriving the normalized value. The RENTPRICE graphed in figure 5 corresponds to an upper bound f**(t) of the optimal debt ratio.

(14) *[f**(t) – mean f**(t)] = [[R(t)/P(t) – mean]*

(14a) *N(f**(t)) = [[(β(t) – β)]]/ σ(β)*

(14b) N[f**(t)] = RENTPRICE

There are a couple of measures of the debt ratio. One measure of the actual debt ratio is L(t)/Y(t) household debt as percent of disposable income. This is series FODSP in FRED. A more useful measure is the debt service ratio i(t)L(t)/Y(t) which measures the *debt burden* graphed in figure 1. This is series TDSP in FRED. The debt L(t) includes all household debt, not just the mortgage debt, because the capital gains led to a general rise in consumption and debt. The *normalized value* of the debt ratio N(f) is equation (15), which is graphed in figure 5 as DEBTSERVICE. This is measured in units of standard deviations from the mean of zero. There is a dramatic deviation above the mean from 1998 to 2005. This sharp rise coincides with the ratio P/Y of housing price index /disposable income, During this period, there was more than a two standard deviation rise in P/Y and a two standard deviation rise in household debt /disposable income.

(15)   N(f) = DEBTSERVICE = [i(t)L(t)/Y(t) – mean]/standard deviation.   .

Compare the DEBT SERVICE with the RENT PRICE in figure 5. This deviation will be used as an upper bound on excess debt.
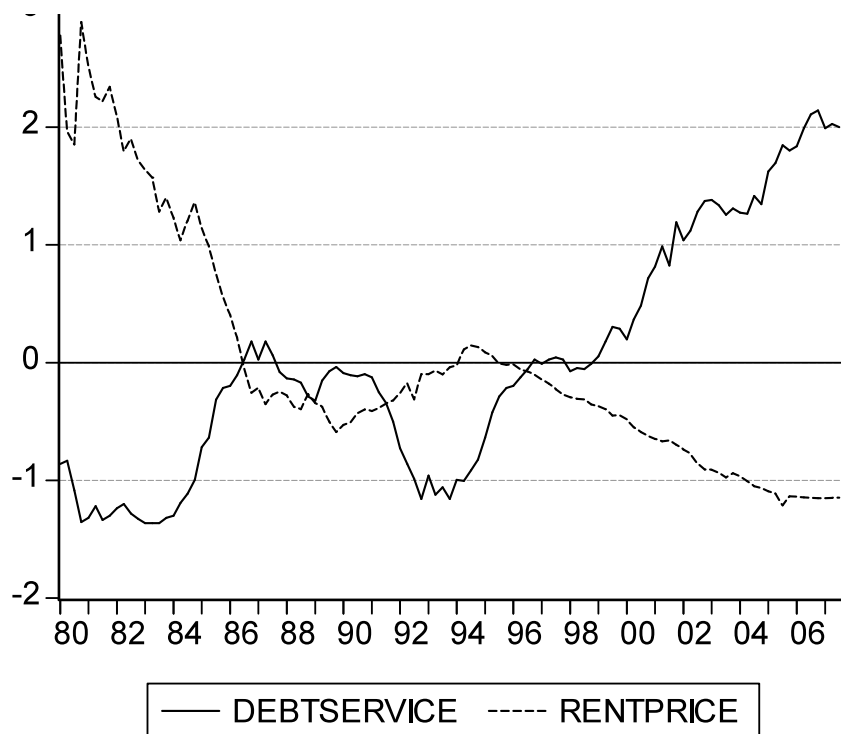
Figure 5.

N[f(t)] = DEBTSERVICE = (household debt service payments as percent of disposable income i(t)L(t)/Y(t) – mean)/standard deviation. N[f**(t)] = RENTPRICE = (rental income/index house price R(t)/P(t) – mean)/standard deviation; Sources FRED, OFHEO

6. Early Warning Signals of the Crisis

My argument in connection with figure 3 is that the excess debt $\Psi(t) = f(t) - f^{**}(t)$ is an early warning signal of a crisis. The next question is how to estimate the excess debt $\Psi(t)$ that corresponds to the models in BOX 1 and is consistent with alternative estimates of the optimal debt.

I estimate excess debt $\Psi(t) = (f(t) - f^{**}(t))$ by using the difference between two normalized variables $N(f) - N(f^{**})$, in figure 5. This difference is *measured in standard deviations*. Debt service is $i(t)L(t)/Y(t)$ household debt service payments as percent of disposable income , figure 1 above, (TDSP in Federal Reserve St. Louis FRED).

(15a)   *$\Psi(t)$ = EXCESS DEBT = N[f(t)] – N[f**(t)]*

        = Normalized {DEBTSERVICE i(t)L(t)/Y(t) – RENTPRICE R(t)/P(t)}.

Excess Debt graphed in figure 6 corresponds to the difference $\Psi(t) = f(t) - f^{**}(t)$ on the vertical axis in figure 3, measured in standard deviations. The probability of a decline in net worth is positively related to $\Psi(t)$ the excess debt. As the excess debt rises, the expected growth declines and the risk increases.

Assume that over the entire period 1980 – 2007 the debt ratio was not excessive. There was no excess debt before 1999. During the period 2000-2004, the high capital gains and low interest rates induced rises in housing prices relative to disposable income and led to rises in the debt service ratio. By 2004-05 the ratio of housing price/disposable income was about three standard deviations above the long-term mean. This drastic rise alarmed several economists who believed that the housing market was drastically overvalued. They were in a minority. It certainly had a negligible effect upon the market for derivatives and the optimism of the "Quants".

The advantages of using excess debt $\Psi(t)$ in figure 6 as an Early Warning Signal compared to just the ratio of housing price/disposable income are that $\Psi(t)$ focuses upon the fundamental determinants of the optimal debt ratio as well as upon the actual ratio.

The excess debt reflects the difference between the debt obligations and the ability to service them. The probability of declines in net worth, the inability of the mortgagors to service their debts and the financial collapse due to leverage - a crisis - are directly related to the excess debt.

Based upon figure 6 early warning signals were given as early as 2003. By 2005, the excess debt was about 2 standard deviations above the mean. Excess debt is not normally distributed, as is obvious from figure 6. A probability cannot be assigned in this case. But whatever the distribution, a 2-3 standard deviation from the mean must be viewed as an Early Warning Signal (EWS). The actual debt was induced by capital gains in excess of the mean interest rate. The debt could only be serviced from capital gains. This situation is unsustainable. When the capital gains fell below the interest rate, the debts could not be serviced. A crisis was inevitable.

The magnitude of the excess debt in figure 6 is a benchmark to evaluate that the economy is being carried away in a bubble. Greenspan, the Fed and IMF had no concept of this and were oblivious to the EWS since 2004.

Figure 6. Early Warning Signal since 2004.

$\Psi(t)$ = EXCESS DEBT = N[f(t)] – N[f**(t)] = DEBTSERVICE i(t)L(t)/Y(t) – RENTRATIO R(t)/Y(t),

7. The Market delusion

A crucial variable in determining the "optimal" debt/net worth in Model I for example is the trend of the housing price. Figure 7 describes the the house price appreciation HPI or capital gains variable CAPGAIN = [P(t) – P(t-1)]/P(t) denoted by $\pi(t)$, which is the four- quarter appreciation of US housing prices, sample 1991q1 – 2011q1.

The Quants/Market estimated the trend of the capital gain on the basis of figures 2 and 4. They assumed that the recent capital gains 1994 -2004 could continue. These values exceeded the mean rate of interest and seemed to offer a "free lunch". My analysis above stresses that optimization should be based upon the assumption that the mean interest rate must exceed the longer run capital gain. This was violated from 1998-2004. Many were aware of that but believed that one could get out "as soon as the music stopped". This was not possible since they all used the same models and had the same data. All could not get out at the same time, when house prices collapsed in 2007. It makes no sense to attribute the collapse to a "black swan".
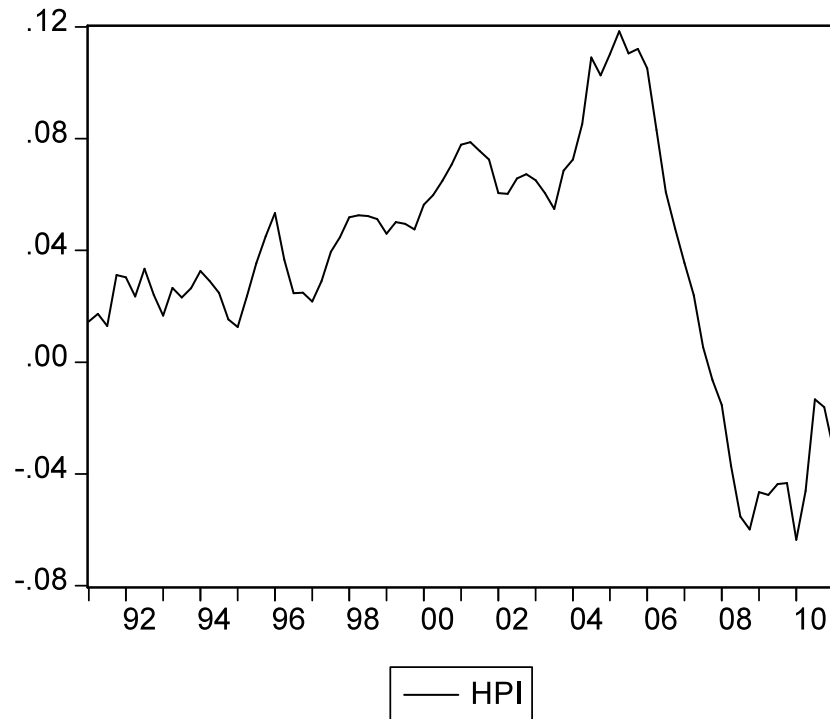
Figure 7. Capital gain CAPGAIN = [P(t) – P(t-1)]/P(t). House Price Index, change over previous 4 quarters (e.g. 0.08 = 8% pa). Federal Housing Finance Industry FHFA USA Indexes. Sample: 1991q1 – 2011q1.

8. The Shadow Banking System: Leverage and Financial Linkages

The financial crisis was precipitated by the mortgage crisis for several reasons. First, a whole structure of financial derivatives was based upon the ultimate debtors – the mortgagors. Insofar as the mortgagors were unable to service their debts, the values of the derivatives fell. Second, the financial intermediaries whose assets and liabilities were based upon the value of derivatives were very highly leveraged. Changes in the values of their net worth were large multiples of changes in asset values. Third, the financial intermediaries were closely linked – the assets of one group were liabilities of another. A cascade was precipitated by the mortgage defaults. Since the "Quants" were following the same rules, the markets would not be liquid.

Charles Prince (former Citigroup CEO) told the FCIC that: "Securitization could be seen as a factory line. As more and more of these subprime mortgages were created as raw material for the securitization process, not surprisingly in hindsight, more and more of it was of lower and lower quality. At the end of that process, the raw material going into it was actually of bad quality, it was toxic quality, and that is what ended up coming out the other end of the pipeline. Wall Street obviously participated in that flow of activity".

A summary of the linkages in the financial system, discussed below, is sketched in BOX 2. My discussion of the role of the shadow banking system is directly based upon the FCIC Report. The origination and securitization of the mortgages relied upon short-term financing from the shadow banking system. Unlike banks and thrifts with access to deposits, investment banks relied more on money market funds and other investors for cash. Commercial paper and repo loans were the main sources. This flood of money and securitization apparently helped boost home prices from the beginning of 2004 until the peak in April 2005, even though homeownership was falling.

BOX 2 Leverages and Linkages in the Shadow Banking System

Households/ Mortgagors => Originator →*Shadow banking System*

(a) Major banks, securitization firms, investment banks: Bear Stearns – vertical integration model, Merrill Lynch, Lehman, Morgan Stanley, Goldman-Sachs, Citigroup, hedge funds. Repackaging of mortgages for sale to investors.

(b) Short-term financing from banks and Money Market Funds: secured by mortgages. Repo, commercial paper

Vulnerabilty of shadow banking system: high leverage, short-term financing, collateral calls, inadequate liquidity. Inadequate information about risks.

i. Mark-to-market reflects House Price Index (HPI). Decline in HPI reduces value of collateral and of net worth.

ii. Contagion. Lenders question value of assets of financial firms and of the collateral posted. Require more/safer collateral. Counterparty risks.

iii. Risk concentrated. High leverage ratios of investment banks up to 40:1, inadequate capital, short term funding, leveraged with commercial paper, repos – using CDOs, Mortgage Backed Assets as collateral.

Leverage (assets/net worth): Citigroup 18:1 (2000) to 32:1 (2007); G-S 17:1 (2000) to 32:1 (2007); Morgan Stanley, Lehman 40:1 (2007)

Revenues, earnings from trading, investing securitization, derivatives:

(a) Revenues: G-S 39% (1997) to 58% (2007); M-L; 42% (1997) to 55% (2005);

(b) Pretax earnings: Lehman 32% (1997) to 80% (2005); Bear Stearns 100% in some years after 2002.

The shadow banking system consisted primarily of investment banks, but also other financial institutions – that operated freely in capital markets beyond the reach of regulatory agencies. Among the largest buyers of commercial paper and repos were the money market mutual funds. Commercial paper was unsecured corporate debt, short-term loans that were rolled over. The market for repos is a market for loans that are collateralized. It seemed like a win-win situation. Mutual funds could earn solid returns and stable companies could borrow more cheaply, Wall St. firms could earn fees putting the deal together. The new parallel banking system, with repos and commercial paper providing better returns for consumers and institutional investors, came at the expense of banks and thrifts.

Over time, investment banks and securities firms used securitization to mimic banking activities outside the regulatory framework. For example, whereas banks traditionally took money from deposits to make loans and held them to maturity, the investment banks used money from the capital market, often from Money Market Mutual Funds, to make loans, package them into securities to sell to investors.

From 2000 to 2007, large banks generally had leverage (assets/net worth) ratios from 16:1 and then up to 32:1 by the end of 2007. Because investment banks were not subject to the same capital requirements as commercial and retail banks, they were given greater latitude to rely on internal risk models in determining capital requirements, and reported higher leverage. At Goldman-Sachs, leverage increased from 17:1 (2001) to 32:1 (2007). Morgan Stanley and Lehman reached 40:1 at the end of 2007. Trading and investments, including securitization and derivative activities, generated an increasing amount of investment banks' revenues and earnings. At G-S, revenues from trading and principal investments increased from 39% (1997) to 58% (2007). At Merrill Lynch revenue from those activities rose from 42% (1997) to 55% (2005). At Lehman, similar activities generated 80% of pre-tax earnings in 2005 up from 32% in 1997. At Bear Stearns, they accounted for 100% of pre-tax earnings in some years after 2002.

Foreign investors sought the high-grade securities but at a higher return than obtained on US Treasuries. They found triple-A assets flowing from the Wall St. securitization machine. As overseas demand drove up prices of securitized debt, it created

an irresistible profit opportunity for the US financial system to engineer "quasi-safe" debt instruments by bundling riskier assets and selling senior tranches. The US housing bubble was financed by large capital inflows.

Some of the key players in the financial network were Lehman, Citigroup, Bear Stearns and Merrill Lynch. Since the late 1990s *Lehman* had built a large mortgage arm, a formidable securities issuance business and a powerful underwriting as well. Lehman pursued a more aggressive growth including greater risk and more leverage. It moved to a "storage" business where Lehman would make and hold longer-term investments. Across both the commercial and residential sectors, the mortgage related assets on Lehman's books increased from $52 billion (2005) to $111 billion (2007). This would lead to Lehman's undoing a year later. Lehman's regulators were aware of this but did not restrain its activities.

As late as July 2007, *Citigroup* was still increasing its leveraged loan business. Citigroup broke new ground in the CDO market. It retained significant exposure to potential losses on its CDO business, particularly with Citibank – the $1 trillion commercial bank whose deposits were insured by the FDIC. In 2005 Citigroup retained the senior and triple-A tranche of most of the CDOs it created. In many cases, Citigroup obtained protection from a monoline insurer. Because these hedges were in place, Citigroup presumed that the risk associated with the retained tranches had been neutralized.

Citigroup reported these tranches at values for which they could not be sold, raising questions about the accuracy of reported earnings. Part of the reason for retaining exposures to super-senior positions in CDOs was their favorable capital treatment. Under the 2001 Recourse Rule, banks were allowed to use their own models to determine how much capital to hold, an amount that varied according to how much the market prices moved. Citigroup judged that the capital requirements for the super-senior tranches of synthetic CDOs it held for trading purposes was effectively zero, because the prices did not move much. As a result, Citigroup held little regulatory capital against the super-senior tranche.

Merrill Lynch focused on the CDO business to boost revenue. To keep its CDO business going, M-L pursued several strategies, all of which involved repackaging riskier mortgages more attractively or buying its own products, when no one else would like to do it. Like Citigroup, M-L increasingly retained for its own portfolio substantial portions of the CDOs it was creating, mainly the super-senior tranches and it increasingly repackaged the hard to sell BBB rated and other low rated tranches into other CDOs. Merrill-Lynch continued to push its CDO business despite signals that the market was weakening.

The bust showed the weaknesses of the system based upon the homeowners' mortgage debt. Early 2007, it became obvious that home prices were falling in regions that had once boomed, and that mortgage originators were floundering and that more and more would be unable to make mortgage payments. The question was how the housing price collapse would affect the financial system that helped inflate the bubble. In theory - held by Greenspan, Bernanke and the market - securitization, over the counter derivatives and the shadow banking system was supposed to distribute risk efficiently among investors. In fact, much of the risk of mortgage backed securities had been taken by a small group of systemically important companies, with huge exposures to senior triple-A tranches of CDOs that supposedly were "super-safe".

As 2007 went on, increasing mortgage delinquencies and defaults compelled the rating agencies to downgrade the mortgage backed securities and then the CDOs. Mortgage delinquencies had hovered around 1% during the early part of the decade jumped in 2005 and kept climbing to 9.7% at the end of 2009. Alarmed investors sent CDO prices plummeting. Hedge funds faced with margin calls from their repo lenders were then forced to sell at distressed prices. Many would shut down. Banks wrote down the value of their holdings by billion of dollars.

By the end of 2008, more than 90% of all tranches of the CDOs had been downgraded. Moody's downgraded nearly all of the Baa CDO tranches. The downgrades were large. More than 80% of Aaa and all of the Baa CDO bonds were eventually downgraded to junk. As market prices of CDOs dropped, *"Mark-to-Market" (M-t-M)* accounting rules require firms to write down their holdings to reflect the lower market

prices. The M-t-M write downs were required on many securities even if there were no actual realized losses, and in some case, even if the firms did not intend to sell the securities.

Determining the market value of securities that did not trade was difficult and subjective and became a contentious issue during the crisis, because the write-downs reduced earning and triggered collateral calls. Large, systemically important firms with significant exposure to these securities would be found to be holding very little capital to protect against potential losses. Most of these companies would turn out to be considered by the authorities as "too big to fail" in the midst of the financial crisis.

When the crises began, *uncertainty and leverage would promote contagion*. Investors realized that they had limited information about the mortgage assets that banks and investment banks held and to which they were exposed. Financial institutions had leveraged themselves with commercial paper, with derivatives and in the short term repo market, in part using CDOs as collateral. Lenders questioned the value of assets that these companies had posted as collateral at the same time that they were questioning the value of those companies' balance sheets.

Bear Stearns (B-S) hedge funds had significant subprime exposure and were affected by the collapse of the housing bubble. The collapse produced pressure on the parent company. The commercial paper and repo markets – the two key components of the shadow banking lending market – quickly reflected the impact of the housing bubble collapse, because of the decline of the value of collateral and concern about the firms' subprime exposure.

In July 2007, B-S hedge funds failed. Its repo lenders – mostly money market mutual funds - required that B-S post more collateral and pay higher interest rates. Mortgage securitization was the biggest piece of B-S most profitable portfolio division. In mortgage securitization, B-S followed a vertically integrated model that made money at every step, from loan origination through securitization and sale. It both acquired and created its own captive organization from originating and securitization/bundling of mortgages into securities that were then sold to investors.

Nearly all hedge funds provide their investors with market value reports, based upon computed M-t-M prices for the fund's various investments. For mortgage backed investments, M-t-M assets was an extremely important exercise, because market values (i) were used to inform investors, (ii) to calculate the hedge fund's total value for internal risk management purposes and (iii) because these assets were held as collateral for repo and other lenders. The crux was that if the value of a hedge fund's portfolio declined, repo and other lenders might require more collateral. Many repo lenders sharpened their focus on the valuation of any collateral with potential subprime exposure. Lenders required increased margins on loans to institutions that appeared to be exposed to the mortgage market. They often required Treasury securities as collateral. Triple-A rated mortgage-backed securities were no longer considered super-safe investments.

Bankers and regulators knew that other investment banks shared B-S weaknesses: leverage, reliance on overnight funding, concentration in illiquid mortgages. As Bear Stearns hedge funds were collapsing, the market viewed Bear Stearns as "the canary in the mine shaft". During all of this Bernanke was not unduly unperturbed. In his testimony to the Joint Economic Committee March, 28, 2007, he said: "The impact on the broader economy and financial markets of the problems in the subprime market seems likely to be contained".

Lehman Brothers was another key player and here is where solvency and liquidity interacted. If assets are greater than liabilities, net worth (capital) is positive and the firm is solvent. If not, the firm is in danger of bankruptcy. In August 2008, shareholder equity was $28 billion. However, balance sheet net worth (capital) is not relevant if the firm is suffering a massive run. If there is a run, and a firm can only get fire sale prices for assets, even large amounts of capital can disappear almost overnight. The chief concerns were Lehman's real estate related investments and its reliance on short term funding, including $7.8 billion of commercial paper and repos as of the end of 2008q1. People were demanding liquidity from Lehman, which it did not have.

The most pressing danger was the potential failure of the repo market. The little regulated repo market grew from an average daily volume $800 billion (2002) to $2.8 trillion (2008). It had become a very deep and liquid market. Even though most borrowers

rolled over repos overnight, it was considered a very safe market because transactions were over collateralized, the ratio of loans/collateral was less than unity. Even long term repo loans had to be unwound every day. To the surprise of both lenders and regulators, as a result to the failure of B-S hedge funds, high quality collateral was not enough to ensure access to the repo market.

If lenders had to sell off large amounts of collateral in order to meet their own cash needs, that action would lead to widespread fire sales of repo collateral and runs by lenders. Most of the Over the Counter derivatives dealers hedge their contracts with offsetting contracts, creating the potential for a series of losses and defaults. This was a *systemic risk* to the financial system as a whole.

In March 2008 the Fed provided a loan to facilitate JP Morgan's purchase of B-S. Bernanke refused to lend to Lehman, because Lehman did not have sufficient collateral. This inconsistency has been the subject of intense debate.

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