

WENDELL H. FLEMING

University Professor Emeritus of Applied Mathematics and Mathematics
Division of Applied Mathematics
Brown University
Providence, RI 02912

Ph.D., 1951, University of Wisconsin

Employment

Rand Corporation 1951–55
Purdue University 1955–58
Brown University 1958–Present
Chairman, Department of Mathematics, 1965–68
Chairman, Division of Applied Mathematics, 1982–85, 1991–94
University Professor 1991 – 95
University Professor Emeritus 1995 –
Professor (Research) 1995 –

Visiting Positions

University of Wisconsin, 1954–54, 1962–63

Stanford University, 1968–69 and Summer 1977

Institut de Recherche d'Informatique et d'Automatique
(one month visits: 1969, 1973, 1974, 1977)

University of Genoa (one month: 1973)

M.I.T., Fall semester 1980

University of Minnesota IMA, Fall semester 1985

University of Minnesota, Ordway Visiting Professor, spring quarter 1993

Honors

NSF Senior Postdoctoral Fellow 1968–69.

Guggenheim Fellow 1976–77.

Invited Plenary speaker, 1982 International Congress of Mathematicians,
in Warsaw. (Postponed until August 1983.)

Fermi Lecturer, Scuola Normale Superiore Pisa, 1986.

Steele Prize, American Mathematical Society, 1987.

Plenary Speaker IEEE Conference on Decision and Control 1988.

Doctor of Science, Honoris Causa, Purdue University, 1991.

Reid Prize, Society for Industrial and Applied Math., 1994.

American Academy of Arts and Sciences, 1995.

Isaacs Award, International Society for Dynamic Games, 2006.

Books

1. *Functions of Several Variables*, Addison-Wesley, 1965, 2nd ed., Springer-Verlag, 1977.
2. *Deterministic and Stochastic Optimal Control*, (with R.W. Rishel), Springer-Verlag, 1975.
3. *Advances in Filtering and Optimal Stochastic Control*, (co-editor with L.G. Gorostiza), Springer Lecture. Notes in Control and Information Sciences, No. 42 (1982).
4. *Recent Advances in Dynamic Programming*, (co-editor with I. Capuzzo-Dolcetta and T. Zolezzi), Springer Lecture Notes in Math, No. 1119, 1985.
5. *Controlled Markov Processes and Nonlinear Evolution Equations*, Accademia Nazionale dei Lincei, Scuola Normale Superiore, 1987.
6. *Stochastic Differential Systems, Stochastic Control Theory and Applications*, (co-editor with P.L. Lions) IMA Volumes in Math. and its Applications No. 10, Springer-Verlag, 1987.
7. *Controlled Markov Processes and Viscosity Solutions*, (with H. M. Soner) Springer-Verlag, 1992; Second Edition, 2006.

Research Publications

1. A Generalized Notion of Boundary, (with L. C. Young), Trans. American Mathematical Society, 76 (1954) 457-484.
2. On a Class of Games over Function Space and Related Variational Problems, Annals of Mathematics, 60 (1954) 578-594.

3. Variational Problems with Constraints, (with R. Bellman and D.V. Widder). *Annali di Mat.* (4), 41 (1956) 301-323.
4. A Note on Differential Games of Prescribed Duration, *Contrib. Theory of Games*, 3, *Annals of Mathematics Studies*, No. 39, pp.407-412.
5. On Differential Games with an Integral Payoff, (with L.D. Berkovitz), *Contrib. Theory of Games*, 3, *Annals of Mathematics Studies* No. 39, pp.413-435.
6. An Example in the Problem of Least Area, *Proc. American Mathematical Society*, 7 (1956) 1063-1074.
7. Representations of Generalized Surfaces as Mixtures, (with L.C. Young), *Rendiconti Circolo Mat. Palermo* (2), 5 (1956) 117-144.
8. Generalized Surfaces with Prescribed Elementary Boundary, (with L.C. Young), *Rendiconti Circolo Mat. Palermo* (2), 5 (1956) 320-340.
9. Functions with Generalized Gradient and Generalized Surfaces, *Annali di Mat. Ser. 4*, 46 (1957), 93-104.
10. Nondegenerate Surfaces of Finite Topological Type, *Trans. American Mathematical Society*, 90 (1959) 323-335.
11. Irreducible Generalized Surfaces, *Rivista Mat. Univ. Parma*, 8 (1957) 251-281.
12. Nondegenerate Surfaces and Fine-Cyclic Surfaces, *Duke Mathematical Journal*, 26 (1959) 137-146.
13. Functions whose Partial Derivatives are Measures, *Illinois Journal of Mathematics*, 4 (1960), 452-478.
14. An Integral Formula for Total Gradient Variation, (with R. Rishel), *Archiv der Math.*, 11 (1960) 218-222.
15. Normal and Integral Currents, (with H. Federer), *Annals of Mathematics*, 72 (1960) 458-520.
16. The convergence Problem for Differential Games, *J. Math. Analysis & Applic.*, 3 (1961) 102-116.
17. On the Oriented Plateau Problem, *Rendiconti Circolo Mat. Palermo* (2), 11, (1962) 1-22.

18. Some Markovian Optimization Problems, *J. of Mathematics & Mechanics*, 12, No. 1 (1963) 131-140.
19. The Convergence Problem for Differential Games II, *Contributions to the Theory of Games*, 5, Princeton University Press, 1964.
20. The Cauchy Problem for Degenerate Quasilinear Parabolic Equations, *J. of Mathematics & Mechanics*, 13 (1964) 987-1008.
21. A Problem of Random Accelerations, Report No. 403, University of Wisconsin Math. Research Center, 1963.
22. Flat Chains Over a Finite Coefficient Group, *Trans. American Math. Society*, 121 (1966) 160-186.
23. Duality and a priori Estimates in Markovian Optimization Problems, *J. Math. Anal. Appl.* 16 (1966), 254-279; Erratum, *ibid.* 19 (1966) 204.
24. On the Existence of Optimal Stochastic Controls, (with M. Nisio), *J. Mathematics and Mechanics* 15 (1966), 777-794.
25. Stochastic Lagrange Multipliers, *Mathematical Theory of Control, Proc. Symp., Univ. of Southern California, 1967*, Academic Press, New York (1967) 443.
26. Optimal Control of Partially Observable Diffusions, *SIAM J. Control* 6 (1968), 194-214.
27. Optimal Continuous Parameter Stochastic Control, *SIAM Review* 11 (1969) 470-509.
28. The Cauchy Problem for a Nonlinear First-Order Partial Differential Equation, *J. Differential Equations* 5(1969) 515-530.
29. Controlled Diffusions Under Polynomial Growth Conditions, in *Calculus of Variations and Control Theory*, A.V. Balakrishnan, ed., Academic Press, New York, 1969, 209-234.
30. Stochastic Control for Small Noise Intensities, *SIAM J. Control* 9 (1971), 473-517.
31. Stochastically Perturbed Dynamical Systems, *Proc. Conf. on Stochastic Differential Equations*, Edmonton, July 1972. *Rocky Mountain Mathematical Journal*, 4 (1974) 407-433.
32. Some One Dimensional Migration Models In Population Genetics Theory, (with C.H. Su). *Theo. Popn. Biol.* 5 (1974) 431-449.
33. Diffusion Processes in Population Biology, *Applied Probability*, 7 (1975) 100-105.

34. A Selection-migration Model in Population Genetics, *J. Math, Biology*, 2 (1975) 219-233.
35. Generalized Solutions in Optimal Stochastic Control, *Proc. Second Kingston Conf. on Differential Games*, Marcel Dekker, 1977.
36. Exit Probabilities and Optimal Stochastic Control, *Applied Math. and Optimization*, 4 (1978) 329-346.
37. Equilibrium Distributions of Continuous Polygenic Traits, *SIAM J. Appl. Math.*, 36 (1979), 148-168.
38. Some Measure-valued Markov Processes in Population Genetics Theory, (with M. Viot), *Indiana Univ. Math. J.*, 28 (1979), 817-844.
39. Measure-valued Processes in the Control of Partially Observable Stochastic Systems, *Applied Math. and Optimization*, 6 (1980) 271-285.
40. Optimal Exit Probabilities and Differential Games, (with C-P Tsai), *Applied Math. and Optimization*, 7 (1981).
41. Optimal Control for Partially Observed Diffusions, (with E. Pardoux), *SIAM J. on Control and Optimization*, 20 (1982) 261-285.
42. Nonlinear Semigroup for Controlled Partially Observed Diffusions, *SIAM. J. on Control and Optimization*, 20 (1982) 286-301.
43. Stochastic Calculus of Variations and Mechanics, *J. Optimiz. Theory Applic.*, 41 (1983) 55-74.
44. Optimal control and Nonlinear Filtering for Nondegenerate Diffusion Processes, (with S.K. Mitter), *Stochastic* 7 (1982) 63-77.
45. Logarithmic Transformations and Stochastic Control, in *Advances in Filtering and Optimal Stochastic Control*, Springer Lecture Notes on Control and Information Sciences No. 42 (1982) 131-142.
46. On Stochastic Relaxed Controls for Partially Observed Diffusions, (with M. Nisio), *Osaka Math. J.* 93 (1984) 71-108.
47. Optimal Control of Markov Processes, *Proc. Intl. Congress of Mathematicians 1983* (invited Plenary Address).
48. A PDE Approach to Asymptotic Estimates for Optimal Exit Probabilities, (with P. E. Souganidis), *Annali della Scuola Normale Superiore Pisa, Ser. IV* 23 (1986) 171-192.

49. Stochastic Variational Formula for Fundamental Solutions of Parabolic PDE, (with S-J. Sheu), *Applied Math and Optimization*, 13 (1985).
50. A Stochastic Control Approach to Some Large Deviations Problems, Proc. Conference on Recent Advances in Dynamic Programming, Rome, March 1984. Springer Lecture Notes in Math. No. 1119, 52-66.
51. An Optimal Stochastic Production Planning Problem with Randomly Fluctuating Demand, (with S.P. Sethi and H.M. Soner), *SIAM J. on Control and Optimization* 25 (1987). 1494-1502.
52. Asymptotic Series and the Method of Vanishing Viscosity, (with P.E. Souganidis), *Indiana Univ. Math. J.* 35 (1986) 425-447.
53. Convex Duality Approach to the Optimal Control of Diffusions, (with D. Vermes), *SIAM J. on Control and Optimization* 27 (1989) 876-907.
54. On the Existence of the Dominant Eigenvalue and its Application to the Large Deviation Properties of an Ergodic Markov Process, (with S. J. Sheu and H.M. Soner), *Stochastics* 22 (1987) 187-199.
55. Value Functions for Two-Player, Zero-Sum Stochastic Differential Games, (with P.E. Souganidis), *Indiana University Math J.* 38 (1989) 293-312.
56. Asymptotic Expansions for Markov processes with Levy Generators, (with H.M. Soner), *Applied Math. and Optimiz.* 19 (1989) 203-223.
57. Generalized solutions and convex duality in optimal control in Partial Differential Equations and the Calculus of Variations, (ed F. Colombini et al), Birkhauser, 1989, pp 461-472.
58. Piecewise monotone filtering with small observation noise, (with E. Pardoux) *SIAM J. on Control and Optimiz.* 27 (1989) 1156-1181.
59. Piecewise monotone filtering in discrete time with small observation noise, (with D. Ji, P. Salame and Q. Zhang) *IEEE Trans. Auto. Control* 36 (1991) 1181-1185.
60. An optimal investment/consumption model with borrowing, (with T. Zariphopoulou) *Math. Oper. Res.* 16 (1991) 802-822.
61. Numerical method for an optimal investment/consumption problem, (with B. Fitzpatrick) in *Math. Oper. Res.* (1991) 823-841.
62. Asymptotic series and exit time probabilities, (with M. James) in *Annals of Probability* 20 (1992) 1369-1384.

63. Nonlinear filtering with small observation noise: piecewise monotone observations, (with Q. Zhang) in *Stochastic Analysis*, eds. E. Merzbach, A. Schwartz and E. Mayer-Wolf. Academic Press (1991) 153–168.
64. Piecewise monotone filtering with small observation noise, (with Q. Zhang) *Proc. Joint US - France Workshop on Stochastic Analysis* (eds. I. Karatzas and D. Ocone) April 1991.
65. Optimal portfolio rebalancing with transactions costs, (with S. Grossman, J-L. Vila and T. Zariphopoulou), preprint.
66. Hedging in incomplete markets with HARA utility, (with D. Duffie, H. M. Soner and T. Zariphopoulou) to appear in the *J. of Economic Dynamics and Control*.
67. Stochastic control and large deviations, *Springer Lecture Notes in Computer Sciences*, No. 653, 1992, pp 291–300. INRIA 25th Anniversary Conference Volume.
68. Risk sensitive control on an infinite time horizon, (with W. M. McEneaney) *SIAM J. on Control and Optimization* 33 (1995) 1881–1915.
69. Optimal investment models and risk sensitive stochastic control, in *IMA Volumes in Math and Applic.* No. 65, 1995 pp 75–88.
70. The risk-sensitive index and the H_2 and H_∞ norms for nonlinear systems, (with M.R. James) *Math. of Control, Signals and Systems* 8 (1995) 199–221
71. Risk sensitive control of finite state machines on an infinite horizon I, (with D. Hernandez-Hernandez) *SIAM J on Control and Optimiz.*, 35 (1997) 1790–1810.
72. Some results and problems in risk sensitive stochastic control, *Computational and Applied Math.* **16** (1997) 99-115.
73. Asymptotics for the principal eigenvalue and eigenvector of a nearly first order operator with large potential, (with S.-J. Sheu) *Annals of Probab.*, 25 (1997), 1953–1994.
74. Risk-sensitive production planning of a stochastic manufacturing system, (with Q. Zhang) *SIAM J. on Control and Optimiz.*, 36 (1998) 1147–1170.
75. Risk sensitive control of finite state machines on an infinite horizon II, (with D. Hernandez-Hernandez), *SIAM J. on Control Optimiz.*, 37 (1999) 1048–1069.
76. Deterministic nonlinear filtering, *Annali Scuola Normale Superiore Pisa, Sci. Fis. Matem. Se 4* 25 (1997) 435–454.

77. A max-plus based algorithm for an HJB equation of nonlinear filtering, (with W.M. McEneaney) *SIAM J. on Control and Optimiz.*, 38 (2000) 683–710.
78. Optimal long term growth rate of expected utility of wealth, (with S.-J. Sheu) *Ann. Appl. Prob.* 9 (1998) 871–903.
79. Deterministic and stochastic approaches to nonlinear filtering, in *System Theory: Modelling, Analysis and Control*, (T.E. Djaferis and I.C. Schick *eds*) Kluwer Academic Publishers, 2000, pp. 121–130.
80. Risk sensitive control and an optimal investment model, (with S.-J. Sheu) *Math. Finance* 10 (2000) 197–213.
81. Stochastic optimal control, international finance and debt, (with J.L. Stein), *J. Banking and Finance* 28 (2004) pp. 979–996.
82. Controlled Markov processes and mathematical finance, in *Nonlinear Analysis Differential Equations and Control*, (ed. F.H. Clarke and R.J. Stein), Kluwer Academic Publishers, NATO Science Series C: Math. Phys. Sci. Vol. 528, (1999), pp. 407–446.
83. Deterministic and stochastic approaches to nonlinear filtering, (with W.M. McEneaney) in *Math. Control Signals Systems*, **14** (2001), pp. 109–142.
84. Risk sensitive control and an optimal investment model II, (with S.-J. Sheu), *Annals Appl. Probab.* 12 (2002) 730–767.
85. Stochastic optimization in discrete time, (with J.L. Stein), in *Economic Theory Dynamics and Markets: Essays in Honor of Ryuzo Sato*, Kluwer Academic Publishers, 2001.
86. Stochastic control models of optimal investment and consumption, *Aportaciones Matematicas*, vol. 16, 2001. Sociedad Matematica Mexicana, pp. 159–203.
87. Max-plus linear partial differential equations, *Research Directions in Distributed Parameter Systems*, (eds. R.C. Smith and M.A. Demetriou) SIAM Publications, 2003, pp. 123–137.
88. Max-plus stochastic processes and control, *Applied Math. Optim.* 49 (2004) 159–181.
89. An optimal consumption model with stochastic volatility, (with D. Hernandez-Hernandez) *Finance and Stochastics*, 7 (2003) 245–262.
90. Optimal investment-consumption models in international finance, (with J.L. Stein) unpublished preprint.

91. An application of stochastic control theory to financial economics, (with T. Pang) *SIAM J. Control Optimiz.* 43 (2004) 502–531.

92. A stochastic control model of investment, production and consumption, (with T. Pang) *Quarterly of Applied Math.* 63 (2005) 71–87.

93. The tradeoff between consumption and investment in incomplete financial markets (with D. Hernandez-Hernandez), *Applied Math. Optim.* 52 (2005) 219–234.

94. Optimal investment models with minimum consumption criteria, *Australian Economic Papers*, December 2005, 307–321.

95. Risk sensitive stochastic control and differential games, to appear in *Communications in Information and Systems*, 6 (2006) 161-178.

Invited Summary Papers Published in Conference Proceedings

1. Optimal Control of Diffusion Processes, in *Functional Analysis and Optimization*, E.R. Caines Ed., Academic Press, (1966) 68–84.

2. Some Problems of Optimal Stochastic Control, in *Stochastic Optimization and Control*, H.F. Karreman Ed., John Wiley, New York (1968) 59-64.

3. Optimal Continuous Parameter Stochastic Control, in *Actes du Congress Int'l. des Math.* 1970, Gauthier-Villars, Paris 3 (1971) 163-167.

4. Nonlinear Partial Differential Equations: Probabilistic and Game Theoretic Methods, Proc. CIME Summer School “Problems in Nonlinear Analysis”, Varenna, August 1970, pp. 95–128.

5. Dynamical Systems with Small Stochastic Terms, in *Techniques of Optimization*, A.V. Balakrishnan, Ed., Academic Press (1972) 325-334.

6. Optimal Control of Diffusion Processes, in *Stochastic Differential Equations*, J.B. Keller, H.P. McKean, Eds., SIMA-AMS Proc., Vol. VI (1973) 163-171.

7. Distributed Parameter Systems in Population Biology, in *Control Theory, Numerical Methods and Computer systems Modeling*, pp. 179-191, A. Bensoussan and J.L. Lions, Eds., *Lecture Notes in Econ. and Math. Systems*, 107 (1975) Springer-Verlag.

8. Some Stochastic Systems Depending on Small Parameters, (with C.P. Tsai), Proc. Intl. Symposium on Dynamical Systems at Brown University, Academic Press (1976) 103-114.

9. Inclusion Probability and Optimal Stochastic Control, *IRIA Seminars Review*, 1977.

10. Optimal Inclusion Probability and Differential Games, (with C.P. Tsai), IRIA Seminars Review, 1977.
11. Some Measure-valued Population Processes, (with M. Viot), in Stochastic Analysis, A. Friedman and M. Pinsky, Eds., Academic Press (1978) 97-108.
12. Optimal Control of Markov Diffusion Processes, Proc. Joint Automatic Control Conference, Vol. 1, October 1978, pp. 355-358.
13. Large Deviations for Diffusions Depending on Small Parameters: A Stochastic Control Method, Proc. 1st AFCET-SMF Symposium, Ecole Polytechnique, September 1978.
14. Minimum Exit Probabilities and Differential Games, (with C.P. Tsai), Proc. 3rd Kingston Conference on Differential Games and Control Theory, Marcel Dekker, 1979.
15. Partially Observed Stochastic Control systems, (with E. Pardoux), Proc. 18th IEEE Conf. on Decision and Control, December 1979, pp. 163-165.
16. Control Optimo de Processos de Diffusion Markovians, (with O. Hernandez Lerma) Conferencias sobre sistemas estocasticas, Ciencia 32 (1981) 39-55.
17. Lecture Notes on Diffusion Approximation and Optimal Stochastic Control, Clemson University, 1979.
18. Stochastic Control Under Partial Observations, 4th Intl. Conf. on Analysis and Optimization of Systems, INRIA, France, December 1980.
19. Optimal Control and Nonlinear Filtering of Nondegenerate Diffusions, (with S.K. Mitter), Proc. 20th IEEE Conf. on Decision and Control, December 1981.
20. A Regular Perturbation Expansion in Nonlinear Filtering, (with R.W. McGwier), Proc. 22nd IEEE Conf. on Decision and Control, December 1983, pp. 82-83.
21. A PDE Approach to Asymptotic Estimates for Optimal Exit Probabilities, (with P. E. Souganidis), Springer Lecture Notes in Control and Information Science, Proc. IFIP Conf., Marseille, March 1984.
22. Asymptotic Series for Solutions to the Dynamic Programming Equation for Diffusions with Small Noise, (with P.E. Souganidis) Proc. 24th IEEE Conf. on Decision and Control, Ft. Lauderdale, Florida, December 11-13, Vol. 1, 1985.
23. A Stochastic Production Planning Problem with Random Demand, (with H.M. Soner) Proc. 24th IEEE Conf. on Decision and Control, Vol. 1, Ft. Lauderdale, Florida, December 11-13, 1985.

24. Generalized Solutions in the Optimal Control of Diffusions (with D. Vermes), Proc. IMA Workshop, June 1986, IMA Vols. in Math. and Applic. No. 10, Springer-Verlag, 1987.
25. Two-Player, Zero-Sum Stochastic Differential Games (with P.E. Souganidis), Analyse Mathematique et Applications, Gauthier-Villars, 1988.
26. Piecewise Linear Filtering With Small Observation Noise (with D. Ji and E Pardoux), Proc. 8th INRIA Conf. on Analysis & Optimiz of Systems, Springer Lecture Notes in Control and Info. Sci Vol. 111 (1988) 725-739.
27. Piecewise Monotone Filtering With Small Observation Noise (with E. Pardoux), Proc. 27th Conf. on Decision and Control, Dec. 1988.
28. Numerical methods for optimal investment-consumption model (with B. Fitzpatrick) Proc. 29th Conf. on Decision and Control, Dec. 1990.
29. Risk sensitive optimal control and differential games (with W. M. McEneaney) Proc. Conf. on Adaptive and Stochastic Control, University of Kansas, 1991, Springer Lecture Notes on Control and Info. Sci. No. 184, 1992, pp. 185–187.
30. Risk sensitive control with ergodic cost criteria (with W. M. McEneaney) Proc. 31st Conf. on Decision and Control, Dec. 1992.
31. Numerical methods for infinite horizon risk sensitive stochastic control (with J. Yang), Proc. 33rd IEEE Conf. on Decision and Control, Dec. 1994.
32. Risk sensitive and robust nonlinear filtering (with W.M. McEneaney) Proc. 36th IEEE Conf. on Decision and Control, Dec. 1997.
33. Risk sensitive control of finite state machines on an infinite horizon I (with D. Hernandez-Hernandez) Proc. 36th IEEE Conf. on Decision and Control, Dec. 1997.
34. Risk sensitive production planning (with Q. Zhang) Proc. 37th IEEE Conf. on Decision and Control, Dec. 1998.
35. An infinite horizon risk sensitive control problem (with S.-J. Sheu), in Optimal Control and Partial Differential Equations (J.L. Menaldi, E. Rofman, A. Sulem eds) IOS Press, 2001, pp. 521–529.
36. Max-plus stochastic control, Proc. Conf. on Stochastic Theory and Control (B. Pasik-Duncan, ed.) Springer LN Control Info. Sci. No. 280, 2002, pp. 111–119.
37. An optimal consumption-investment problem for factor-dependent models, (with Daniel Hernandez-Hernandez) Proc. Conf. on Stochastic Theory and Control (B. Pasik-Duncan ed.) Springer LN Control Info. Sci. No. 280, 2002, pp. 121–130.

38. Some optimal investment, production and consumption models, Proc. Snowbird Math. Of Finance Conf. (G. Yin and Q. Zhang eds.) American Math. Soc. Contemporary Math. No. 351, 2003, pp. 115–123.

39. Max-plus approaches to continuous space control and dynamic programming, in Idempotent Mathematics and Mathematical Physics (G.L. Litvinov and V.P. Maslov eds.) American Math. Soc. Contemporary Math. No. 377, 2005, pp. 145–160. February 2006.