Applied Math 166 Problem Set 10 5/2/03

13.9 H_0: $\mu_1 = \mu_2 = \mu_3 = 0$ vs. H_a: One or more of μ_i's differ

a. $y_{1\cdot} = 14(.93) = 13.02$; $y_{2\cdot} = 14(1.21) = 16.94$; $y_{3\cdot} = 14(.92) = 12.88$

b. Total $= y_{1\cdot} + y_{2\cdot} + y_{3\cdot} = 42.84$

c. $\text{CM} = \frac{(42.84)^2}{42} = 43.6968$

d. $\text{SST} = \frac{(13.02)^2+(16.94)^2+(12.88)^2}{14} - \text{CM} = .7588$

e. $s_1^2 = 14(.04)^2 = .0224$; $s_2^2 = 14(.03)^2 = .0126$; $s_3^2 = 14(.04)^2 = .0224$

f. & g. $\text{SSE} = \sum_{i=1}^{p} (n_i - 1)s_1^2 = 13(.0224) + 13(.0126) + 13(.0224) = .7462$

h.

Source	df	SS	MS	F
Treatments	2	.7588	.3794	19.83
Error	39	.7462	.019133	
Total	41			

i. The test statistic is

$F = \frac{\text{MST}}{\text{MSE}} = \frac{.3794}{.019133} = 19.83$

and the rejection region with 2 and 39 df is approximately $F > F_{.05} = 3.23$. The null hypothesis is rejected and there is a difference between the means.

p-value < 0.005.

13.10a. Calculate

(1) $\text{CM} = \frac{\left[\sum_i \sum_j y_{ij}\right]^2}{n} = \frac{573,306.4}{36} = 15,925.178$

(2) $\text{TSS} = \sum_i \sum_j y_{ij}^2 - \text{CM} = 16,160.397 - \text{CM} = 235.219$

(3) $\text{SST} = \sum_i \frac{y_{i\cdot}^2}{n_i} - \text{CM} = \frac{(287.58)^2+(245.56)^2+(224.03)^2}{12} - \text{CM} = 174.106$

(4) $\text{SSE} = \text{TSS} - \text{SST} = 61.113$

The ANOVA table is shown below. The F statistic is

$F = \frac{\text{MST}}{\text{MSE}} = 47.007$

Source	d.f.	SS	MS
Treatments	2	174.106	87.053
Error	33	61.113	1.852
Total	35	235.219	

From Table 7, $F = 47.007 > 6.266$, so the p-value $< .005$. We reject H_0 when $\alpha = .01$. Note that the value 6.266 was found by interpolation $\left[6.266 = 6.35 - \left(\frac{3}{10}\right)(6.35 - 6.07)\right]$.

b. We must assume that we have normally distributed data with a common variance. Also we assume that the values from low, medium, and high concentrations of acetonitrile are normally distributed with common variance.

13.11 Using the pooled sum of squares formula for SSE, we have

$$\text{SSE} = \sum (n_i - 1)s_i^2 = 44(.7)^2 + 101(.64)^2 + 17(.9)^2 = 76.6996$$

The treatment totals are

$$y_{1\cdot} = 45(4.59) = 206.55 \qquad y_{2\cdot} = 102(4.88) = 497.76 \qquad y_{3\cdot} = 18(6.24) = 112.32$$

so that

$$\text{CM} = \frac{[\sum\sum y_{ij}]^2}{n} = \frac{(816.63)^2}{165} = 4041.725$$

$$\text{SST} = \sum \frac{y_{i\cdot}^2}{n_i} - \text{CM} = \sum n_i \bar{y}_{i\cdot}^2 - \text{CM} = 45(4.59)^2 + 102(4.88)^2 + 18(6.24)^2 - \text{CM}$$
$$= 4078.010 - 4041.725 = 36.285$$

The ANOVA table is shown below. Then

$$F = \frac{\text{MST}}{\text{MSE}} = \frac{18.143}{.4735} = 38.316$$

Source	d.f.	SS	MS
Treatments	2	36.285	18.143
Error	162	76.6996	0.4735
Total	164		

From Table 7, $F = 38.316 > 7.88 = F_{.005}$ for 2 and ∞ degrees of freedom. Thus, the p-value is less than .005. We reject H_0 at the $\alpha = .05$ level (p-value $< .005$).

13.21 Need to consider

$$(\overline{Y}_{1\cdot} - \overline{Y}_{2\cdot}) \pm (t_{.025,39})s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

or

$$s = \sqrt{\text{MSE}} = .1383.$$

We give as our 95% estimate interval

$$(.93 - 1.21) \pm 1.96(.1383)\sqrt{\frac{2}{14}}$$

or

$$-.28 \pm .102 = (-.382, -.178).$$

At the 95% confidence level, we would conclude that there is a significant difference between the mean bone densities for the two groups of women since the confidence interval formed contains all negative values. This suggests that $\mu_2 > \mu_1$.

13.24a. A 95% confidence interval for μ_L is

$$\overline{y}_L \pm t_{.025,33} \frac{s}{\sqrt{n_L}} = 23.965 \pm 1.96\sqrt{\frac{1.852}{12}} = 23.965 \pm .77$$

b. A 90% confidence interval for $\mu_L - \mu_M$ is

$$(\overline{y}_L - \overline{y}_M) \pm t_{.05,33} \frac{s}{\sqrt{\frac{1}{n_L}+\frac{1}{n_M}}}$$

or

$$(23.965 - 20.463) \pm 1.645\sqrt{1.852}\sqrt{\frac{1}{12}+\frac{1}{12}} = 3.502 \pm .914$$

13.25a. A 95% confidence interval for μ_B is

$$\overline{y}_B \pm t_{.025,162} \frac{s}{\sqrt{n_B}} = 6.24 \pm 1.96\sqrt{\frac{.4735}{18}} = 6.24 \pm .318$$

b. A 95% confidence interval for $\mu_s - \mu_L$ is

$$(\overline{y}_s - \overline{y}_L) \pm t_{.05,162} \frac{s}{\sqrt{\frac{1}{n_s}+\frac{1}{n_L}}}$$

or

$$(4.59 - 4.58) \pm 1.96\sqrt{.4735}\sqrt{\frac{1}{45}+\frac{1}{102}} \quad \text{or} \quad -.29 \pm .241$$

c. Probably not. The driving habits of people vary from town to town. Thus, the vehicles sampled do not represent a random sample of vehicles from all towns.

13.27a. We wish to test H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$ against the general alternative. Calculate

$\text{CM} = \frac{(49)^2}{16} = 150.0625$

$\text{TSS} = 183 - \text{CM} = 32.9375$

$\text{SST} = \frac{(8)^2+(12)^2+(13)^2+(16)^2}{4} - \text{CM} = 8.1875$

The ANOVA table is shown below. Then

$F = \frac{\text{MST}}{\text{MSE}} = 1.32$

Source	d.f.	SS	MS
Treatments	3	8.1875	2.7292
Error	12	24.7500	2.0625
Total	15	32.9375	

which is compared to $F_{.05} = 3.49$ with 3 and 12 degrees of freedom. We do not reject H_0.

b. $(\overline{y}_{4\cdot} - \overline{y}_{1\cdot}) \pm t_{.025,12}\sqrt{s^2\left(\frac{1}{n_4}+\frac{1}{n_1}\right)}$ or $2 \pm 2.179\sqrt{2.0625\left(\frac{2}{4}\right)}$ or 2 ± 2.21

or

$$-.21 < \mu_4 - \mu_1 \le 4.21.$$

Intervals constructed in this manner will enclose $(\mu_4 - \mu_1)$ 95% of the time in repeated sampling. Hence we are fairly confident that this particular interval encloses $(\mu_4 - \mu_1)$.

13.37 a. A summary of the data is

$Y_{1\cdot} = 9.32$ $Y_{2\cdot} = 9.45$ $Y_{\cdot 1} = 2.27$

$Y_{\cdot 2} = 3.45$ $Y_{\cdot 3} = 2.14$ $Y_{\cdot 4} = 3.73$

$Y_{\cdot 5} = 2.93$ $Y_{\cdot 6} = 4.35$

(4)

$\sum_{i=1}^{2} \sum_{j=1}^{6} y_{ij} = 18.77$

$\sum_{i=1}^{2} \sum_{j=1}^{6} y_{ij}^2 = 31.1013$

$\text{CM} = \frac{(18.77)^2}{12} = 29.3594$

$\text{Total SS} = 31.1013 - 29.3594 = 1.7419$

$\text{SST} = \frac{(9.32)^2+(9.45)^2}{6} - 29.3594 = .0014$

$\text{SSB} = \frac{(2.27)^2+(2.45)^2+(2.14)^2+(3.73)^2+(2.93)^2+(4.25)^2}{2} - 29.3594$
$= 1.7382$

$\text{SSE} = 1.7419 - .0014 - 1.7382 = .0023$

The ANOVA table is

Source	df	SS	MS
Computer	1	.0014	.0014
Program	5	1.7382	.3476
Error	5	.0023	.00045
Total	11	1.7419	

To test $H_0\colon \mu_1 = \mu_2$, we use

$F = \frac{\text{MST}}{\text{MSE}} = 3.05$

Since $3.05 < 6.61 = F_{.05}$ with 1 and 5 degrees of freedom, we fail to reject H_0. Thus, we see no evidence of a difference in mean CPU time between computer 1 and computer 2.

This decision is the same as the one reached in Exercise 12.10**a**.

b. The p-value is greater than .10. This is consistent with Exercise 12.10**b**.

c. Except for round off, $S_D^2 = 2\text{MSE}$.

13.40 Calculate

$\text{CM} = \frac{(52.333)^2}{20} = 136.937$ $\text{TSS} = 138.603 - \text{CM} = 1.666$

$\text{SST} = \frac{549.558}{4} - \text{CM} = .452$ $\text{SSB} = \frac{689.943}{5} - \text{CM} = 1.052$

The ANOVA table is shown at the right.

Source	d.f.	SS	MS
Treatments	4	.452	.113
Blocks	3	1.052	.3507
Error	12	.162	.0135
Total	19	1.666	

a. The F statistic to detect a difference due to varieties has 4 and 12 degrees of freedom:

$$F = \frac{\text{MST}}{\text{MSE}} = 8.37$$

Since $8.37 > 6.52$, the p-value $< .005$, and we reject H_0 at the $\alpha = .01$ level.

b. The F statistic for blocks, with 3 and 12 degrees of freedom, is

$$F = \frac{\text{MSB}}{\text{MSE}} = 25.97$$

Since $25.97 > 3.49 = F_{.05}$, we reject H_0: there is evidence to suggest a significant difference due to blocks.

13.41 Using a randomized block design with locations as blocks, recall that $\text{CM} = 150.0625$, $\text{TSS} = 32.9375$, and $\text{SST} = 8.1875$. Then

$$\text{SSB} = \frac{(15)^2+(8)^2+(14)^2+(12)^2}{4} - \text{CM} = 7.1875$$

SSE will be obtained by subtraction; see the ANOVA table. To test for a difference in treatment means, the test statistic is

Source	d.f.	SS	MS
Treatments	3	8.1875	2.729
Blocks	3	7.1875	2.396
Error	9	17.5625	1.95139
Total	15	32.9375	

$F = \frac{\text{MST}}{\text{MSE}} = 1.40$

which is compared to $F_{.05} = 3.86$ with 3 and 9 degrees of freedom. There is no evidence of significant treatment differences. Notice that the test for block differences yields $F = \frac{\text{MST}}{\text{MSE}} = 1.23$, which is not significant. That is, the evidence supports the hypothesis that blocking was not worthwhile.