1. Telegraphic signals "dot" and "dash" are sent in the proportion 3:4. Owing to a very noisy transmission channel, a dot becomes a dash with probability 1/4, whereas a dash becomes a dot with probability 1/3.

(a) If a dot is received, what is the probability that it had been sent as a dot?

(b) If five successive dots are received, what is the pdf of the rv X that counts the number of dots that were sent?

Let T denote the event "dot sent" and let H denote the event "dash sent"

$P(T)$ and $P(H)$ have relative magnitudes 3:4, so

$P(T) = 3/7$ and $P(H) = 4/7$.

Let R_T denote the event "dot Received" and let R_H denote the event "dash Received"

From the given information, $P(R_H \mid T) = 1/4$ (and $P(R_T \mid T) = 3/4$)

and $P(R_T \mid H) = 1/3$ (and $P(R_H \mid H) = 2/3$).

(a) Find $P(T \mid R_T)$. By Bayes Formula,

$$P(T \mid R_T) = P(R_T \mid T)P(T) \Big/ \left[P(R_T \mid T)P(T) + P(R_T \mid H)P(H) \right]$$

$$= \left(\frac{3}{4}\cdot\frac{3}{7}\right) \Big/ \left[\frac{3}{4}\cdot\frac{3}{7} + \frac{1}{3}\cdot\frac{4}{7}\right] = \frac{9}{4} \Big/ \left[\frac{9}{4} + \frac{4}{3}\right] = \frac{9}{4} \Big/ \frac{43}{12} = \underline{\underline{\frac{27}{43}}}$$

(b) This fits the situation for the binomial distribution, where "success" means "dot sent" and we have identical conditions "dot Received"

X is binomial with $\underline{n=5}$ and $p = P(T \mid R_T) = \frac{27}{43} = \underline{0.628}$.

2. Four shoes are taken at random from a dark closet that contains five pairs of shoes. What is the probability that there is at least one pair among them?

To have a pair, we need to select one right shoe and a left shoe from the same pair. Or, equivalently, in order not to have a pair, the left shoes that we select must come from different pairs than the right shoes that we select.

Use the sample point method. The number of ways that we can choose a combination of 4 shoes from a population of 10 shoes is $\binom{10}{4} = \frac{10\cdot 9\cdot 8\cdot 7}{4\cdot 3\cdot 2\cdot 1} = 210$.

Let A denote the event "a pair is selected". It is easier to count the sample points in A^c, "no pair is selected".

We might choose 4 left shoes: $\binom{5}{4}\binom{1}{0}$ ways

3 left shoes and one right shoe: $\binom{5}{3}\binom{2}{1}$ ways

2 left shoes and 2 right shoes from different pairs: $\binom{5}{2}\binom{3}{2}$ ways

1 " " " 3 " " " " " : $\binom{5}{1}\binom{4}{3}$ ways

0 " " " 4 " " " " " : $\binom{5}{0}\binom{5}{4}$ ways.

$$\therefore P(A) = 1 - \left\{\binom{5}{4}\binom{1}{0} + \binom{5}{3}\binom{2}{1} + \binom{5}{2}\binom{3}{2} + \binom{5}{1}\binom{4}{3} + \binom{5}{0}\binom{5}{4}\right\} \Big/ \binom{10}{4}$$

$$= 1 - \{5 + 20 + 30 + 20 + 5\}/210 = 1 - 80/210 = \underline{13/21}.$$

3. A 300 page book contains 200 misprints.
(a) What is the probability that more than one misprint occurs on page 151?
(b) What is the probability of no misprint on pages 1 through 10?

One can formulate this in a way for the binomial distribution to apply (e.g., each keystroke is a Bernoulli trial, there are n keystrokes on a page, ---) and then use a Poisson approximation of the binomial since n is large and p is small.

Or one can formulate this in terms of the Poisson distribution directly.

(a) Per page, there are on average $200/300 = 2/3 = 0.667$ errors.

r.v. X = # misprints on page 151 Poisson $\lambda = 2/3$.

$$P(X>1) = 1 - P(X\le 1) = 1 - P(X=0) - P(X=1)$$
$$= 1 - e^{-\lambda} - \lambda e^{-\lambda} = 1 - (1+\lambda)e^{-\lambda}$$
$$= 1 - (5/3)e^{-2/3} = \underline{\underline{0.1443}}$$

(b) Let Y = # misprints on pages 1-10. Poisson $\lambda = 10 \cdot 2/3 = 20/3$

$$P(Y=0) = e^{-\lambda} = e^{-20/3} = \underline{\underline{0.0013}}$$

4. Suppose a regular 4-sided die with faces numbered 1, 2, 3 and 4 is rolled and the number Y on the face on which it lands is recorded. Then Y fair coins are tossed and the number X of heads that occur is counted.

(a) What is the pdf of X?

(b) If we observe $(X = 1)$, what is the (conditional) pdf of Y, i.e. $p(y) = P(Y = y|X = 1)$?

(a) Use the total probability formula to compute $P(X=x)$ for $x = 0, 1, 2, 3, 4$

$$P(X=x) = \sum_{y=1}^{4} P(X=x \mid Y=y)\, P(Y=y)$$

	$x = 0$	1	2	3	4	
$Y=1$:	$1/2$	$1/2$				$\times\ 1/4$
$Y=2$:	$1/4$	$1/2$	$1/4$			$\times\ 1/4$
$Y=3$:	$1/8$	$3/8$	$3/8$	$1/8$		$\times\ 1/4$
$Y=4$:	$1/16$	$1/4$	$3/8$	$1/4$	$1/16$	$\times\ 1/4$
$P(X=x)$:	$15/64$	$26/64$	$16/64$	$6/64$	$1/64$	

(b) $p(y) = P(Y=y \mid X=1) = P(Y=y \cap X=1) / P(X=1) = \dfrac{P(X=1 \mid Y=y) P(Y=y)}{P(X=1)}$

$p(1) = \dfrac{\frac{1}{2} \cdot \frac{1}{4}}{\frac{26}{64}} = \dfrac{64}{8 \cdot 26} = \underline{\underline{\dfrac{4}{13}}}$; $p(2) = \dfrac{\frac{1}{2} \cdot \frac{1}{4}}{\frac{26}{64}} = \underline{\underline{\dfrac{4}{13}}}$; $p(3) = \dfrac{\frac{3}{8} \cdot \frac{1}{4}}{\frac{26}{64}} = \underline{\underline{\dfrac{3}{13}}}$

$p(4) = \dfrac{\frac{1}{4} \cdot \frac{1}{4}}{26/64} = \underline{\underline{\dfrac{2}{13}}}$

5. In a marketing research test, 25 males were asked to shave one side of their face with one brand of razor blade and the other side with a second brand. They were to use the blades for seven days and then decide which was giving the smoother shave. Suppose that 18 of the subjects prefer blade A.

(a) If there is indeed no difference between the two blades, what kind of a distribution should the count X of those expressing a preference for A have?

(b) If there is no difference between the two blades, what is the probability that 18 or more of the subjects would express a preference for blade A and what is the probability that $|X - 12.5| \geq 5.5$? (How convincing is the evidence that the blades are really different?)

(a) X counts "successes" (prefer A) in 25 independent identical trials,

So X is binomial, $n=25$, $p=0.5$

(b) $P(X \geq 18) = 1 - P(X \leq 17) = 1 - 0.978$ (from Table 1)

$= 0.022.$

$$P(|X-12.5| \geq 5.5) = P(X \geq 18 \cup X \leq 7)$$

$$= P(X \geq 18) + P(X \leq 7)$$

$$= 0.022 + 0.022 \quad \text{(from Table 1)}$$

$$= 0.044 \quad \underline{\underline{\text{small}}}$$

The evidence is reasonably convincing that the blades are really different.